Tests of Gravity in the Solar System

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Outline

• Introduction / Motivation
• Experimental confirmation of GR with space tests
• Space tests for gravitational physics – an overview
• Unexplained phenomena / Observations in the solar system
• Anomalies from satellite tracking
  • Pioneer Anomaly
    • Present status of analysis
    • Thermal models
    • Other attempts for explanation
  • Fly-by Anomaly
  • GRACE Anomaly
• Outlook
Motivation

Theoretical Background
- Standard Model of Quantum Physics
- Special Relativity
- General Relativity
- Statistical Physics

But:
- Incompatibility of Quantum Theory and Gravitation

Experimental proofs
- Experimental confirmation of the Standard Model makes quantization of space and time a likely approach
- Gravitational theory well confirmed by experiments and observations
- Precision Cosmology (COBE, W-MAP, etc.)

Unexplained phenomena?
- Space – a unique laboratory
- Do we see effects on satellites in deep space orbits?
- Is gravity modified on larger scales?
Experimental confirmation of GR

Foundations:
- Universality of Free Fall
- Local Lorentz Invariance
- Universality of Gravitational Redshift

Predictions:
- Solar System Effects
  - Perihelion shift
  - Gravitational Redshift
  - Light deflection
  - Time delay
  - Gravitomagnetic effects
- Strong field observations
  - Binary systems
  - Black holes
- Gravitational waves
- Cosmology

Tests in the Post-Newtonian frame

\[ g_{00} = -1 + 2\alpha \frac{U}{c^2} - 2\beta \frac{U^2}{c^4} \]

\[ g_{0i} = 4\mu \frac{(\vec{J} \times \vec{r})_i}{c^3 r^3} \]

\[ g_{ij} = (1 + 2\gamma) \frac{U}{c^2} \]

<table>
<thead>
<tr>
<th>Effect</th>
<th>Method</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>perihelion shift</td>
<td>astronomical observations</td>
<td>(</td>
</tr>
<tr>
<td>light deflection</td>
<td>Very Long Baseline Interference</td>
<td>(</td>
</tr>
<tr>
<td>time delay</td>
<td>Cassini S/C</td>
<td>(</td>
</tr>
<tr>
<td>gravitational redshift</td>
<td>Gravity Probe A</td>
<td>(</td>
</tr>
<tr>
<td>Lense-Thirring effect</td>
<td>LAGEOS satellites</td>
<td>(\leq 0.1)</td>
</tr>
<tr>
<td>Schiff effect</td>
<td>Gravity Probe B</td>
<td>(</td>
</tr>
</tbody>
</table>

(not yet confirmed)
Space Tests for GR

Satellite experiments determining the Eddington Parameters
Shapiro Time Delay

Einstein-Infeld-Hoffmann Equation

- Numerical models based on an isotropic PPN - n-body metric
- Planets and asteroids considered to be point masses
- Accelerations calculated wrt the barycentre of the solar system.

\[
\ddot{r}_i = \sum_{j \neq i} \frac{Gm_j(r_j - r_i)}{|r_j - r_i|^3} \\
\left[ 1 - \frac{2(\beta + \gamma)}{c^2} \sum_{k \neq i} \frac{Gm_k}{|r_k - r_i|} \right] - \frac{2\beta - 1}{c^2} \sum_{k \neq j} \frac{Gm_k}{|r_j - r_k|^3} + \gamma \frac{\dot{r}_i^2}{c^2} + (1 + \gamma) \frac{\dot{r}_j^2}{c^2} - \frac{2 + 2\gamma}{c^2} \dot{r}_i \cdot \dot{r}_j - \frac{3}{2c^2} \left( \frac{\dot{r}_i \cdot \dot{r}_j}{|r_j - r_i|} \right)^2 + \frac{1}{c^2} (\ddot{r}_j - \ddot{r}_i) \cdot \ddot{r}_j
\]

Time delay in space time curved by sun and earth

\[
\Delta t = \frac{1}{c} \left[ \left( \dot{r}_i^C - \dot{r}_S^C \right) + (1 + \gamma)Gm_S \ln \left( \frac{r_s^S + r_i^S + \dot{r}_i^S - \dot{r}_s^S + (1 + \gamma)Gm_s / c^2}{r_s^S + r_i^S - \dot{r}_i^S - \dot{r}_s^S + (1 + \gamma)Gm_s / c^2} \right) + (1 + \gamma)Gm_E \ln \left( \frac{r_s^E + r_i^E + \dot{r}_i^E - \dot{r}_s^E}{r_s^E + r_i^E - \dot{r}_i^E - \dot{r}_s^E} \right) \right]
\]

Cassini Conjunction Experiment (Bertotti et al. 2002):

- Satellite – Earth distance: > 10^9 km
- Ranging: X~7.14GHz & Ka~34.1GHz (dual band)
- Result: \( \gamma = 1 + (2.1 \pm 2.3) \times 10^{-5} \)
Gravitational redshift

- Gravity Probe A experiment (Vessot et al., 1974): ballistic rocket flight
- Confirmation of the Universality of the Gravitational Redshift: All clocks run the same and experience the same frequency shift in gravitational fields— independent of their physical characteristics
- GP-A: comparison of H-masers on earth and in a capsule on a ballistic flight path
- Accuracy \( \delta \left( \frac{f_1}{f_2} \right) \leq 10^{-4} \)

\[ \frac{f_1}{f_2} = \frac{k(u_1)}{k(u_2)} = \sqrt{\frac{g_{00}(x_1)}{g_{00}(x_2)}} \left( 1 - \frac{U(x_1) - U(x_2)}{c^2} \right) \]
Lunar Laser Ranging

Retroreflectors on moon surface: Installed between 1969 and 1973 with Apollo 11, 14, 15 and Luna 17, 21

Resolution: ca. 2 cm (< 1 cm)
Laser pulse width: 150 – 300 ps
Pulse frequency: 10 Hz
Illuminated area on moon surface: 20 km²
1 of $10^{19}$ photons observed (1 photon per 10 pulses)
Lunar Laser Ranging

Earth and moon are of different chemical composition and freely falling in the sun’s gravitational field. This enables to perform tests of the Weak Equivalence Principle by ranging between earth and moon.

\[ \eta_E \leq 10^{-13} \quad \text{for} \quad a_d = \eta_E \frac{Gm_S}{r_{ES}^2} \]

Dependent on the validity of the Strong Equivalence Principle:

Does self-gravitation \( \Omega \) contributes the same to inertial and gravitational mass?

\[
a_d = -\frac{m_{total}^\prime}{r_{EM}^3} \frac{r_{EM}}{r_{EM}^3} + \eta \left( \left( \frac{\Omega}{m} \right)_E - \left( \frac{\Omega}{m} \right)_M \right) \frac{m_S^r r_{ES}^r}{r_{ES}^3} + \frac{m_{\text{passive}}}{m} \frac{m_S}{r_{ES}^3} \frac{r_{ES}^3 - r_{MS}^3}{r_{MS}^3}
\]

Nordtvedt parameter \( \eta \leq 4 \beta - \gamma - 3 - \frac{10}{3} \xi - \alpha_1 + \frac{2}{3} \xi_1 - \frac{2}{3} \xi_2 - \frac{1}{3} \xi_2 \leq 10^{-3} \)
Gravity Probe B

- Precise measurement of gyro prececcions due to space-time curvature of the rotating earth.
- Geodetic Precession: 6.6 arcsec per year
- Lense-Thirring precession (frame dragging, Schiff effect):
  0.042 arcsec pro Jahr für die Lense-Thirring-Präzession
- Experiment already proposed 1959 (!) (G. Pugh)
- Carried out: April 2004 to August 2005

\[
\frac{d\tilde{S}}{dt} = \tilde{\Omega} \times \tilde{S} = \left(-\frac{1}{2} \tilde{v} \times \tilde{a} + \frac{3}{2} \tilde{v} \times \nabla U + \nabla \times \tilde{h}\right) \times \tilde{S}
\]
Precise gyroscopes

- Ideally round spheres made from fused silica, Nb-coated for superconductivity: deviation from sphericity max. 5 nm
- Electrostatic levitation, frictionless bearing: spin-rate change: 1 % in 1000 years
- Accuracy: \(10^{-11}°/\text{h}^{-1}\) (1 revolution in 11.5 billion years)

![Diagram of a hair from 16 km distance]
Precision-thruster for satellites

- Drag free control: Satellit follows a freely falling test mass on a geodetic
- Closed-loop control of test mass movement
- Micropropulsion thrusters guide the satellite
- Field Emission Electrical Propulsion (FEEP)

Residual acceleration: ca. $10^{-14}$ m / s$^2$
Thrust-increment resolution: ca. 0.1 µN
Specific impulse: > 10,000 s
WEP tests on satellites

\[ \text{dr} = \frac{1}{3} r_1 \eta_E \approx 10^{-12} \text{m} \]

Orbit radius difference too small for direct measurement

For weak mechanical coupling both test masses as well as the satellite form a spring-mass system, which amplitude varies periodically with orbit frequency. Can be measured with high precision.
MICROSCOPE

Micro-satellite à Trainée Compensée pour l’Observation du Principe d’Équivalence

Missions parameter:
- Sunsynchronous orbit: 660 km
- Orbit exzentricity: $< 5 \cdot 10^{-3}$
- Spin rate: varying for modulation of the orbit-frequency
- Signal frequenz: $(\pi +1/2) f_{orb}$ und $(\pi +3/2) f_{orb}$
- Missions duration: 6 to 12 months
- Satellite mass: $< 120$ kg

x-axis: sensitive axis
z-axis: satellite spin axis

ONERA
Clocks in space: ACES / PHARAO

- PHARAO most precise clock in space
- Allan-Variance: $10^{-16}$
- ACES enables Phase and Frequency comparison between a Cs-atomic clock and earth bound clocks:
  - 0.3 ps during ISS pass (~5min.)
  - 7 ps during 1 day
  - 23 ps during 10 days

<table>
<thead>
<tr>
<th>Effect</th>
<th>Precision</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal Doppler effect</td>
<td>$10^{-2}$ s per day</td>
</tr>
<tr>
<td>Transversal Doppler effekt</td>
<td>$10^{-5}$ s per day</td>
</tr>
<tr>
<td>Sagnac effekt</td>
<td>$3 \cdot 10^{-7}$ s per orbit</td>
</tr>
<tr>
<td>1st ord. Gravitational redshift</td>
<td>$8 \cdot 10^{-5}$ s per day</td>
</tr>
<tr>
<td>2nd ord. Gravitational redshift</td>
<td>$10^{-13}$ s per day</td>
</tr>
<tr>
<td>Gravitational time delay</td>
<td>$4 \cdot 10^{-11}$ s</td>
</tr>
<tr>
<td>Gravitomagnetic clock effect</td>
<td>$10^{-7}$ s per orbit</td>
</tr>
</tbody>
</table>

Resolution of the time-link:
- common view: 1 ps
- non-common view: 3 ps for $\Delta t < 1,000$ s
  10 ps for $\Delta t < 10,000$ s
Time transfer (MWL)

- MWL (Microwave link to ISS) developed for transfers of time signals between ISS and earth
- Frequency comparison with a relative accuracy of $10^{-16}$ (230 fs per pass, 5 ps per orbit)
- 2 symmetric 1-way links for a continuously pseudo-noise coded signal
- High Ku-band chip rate (100 MChip/s) to improve the resolution and to depress possible multipath signals
- 1 W power (S- and Ku-band)
- Enables additional ranging with $\lambda/1,000 = 24 \mu$m accuracy.
Optical links for tracking

- Optical transponder (On-board-laser, telescope, on-board.clock)
- Successfully demonstrated over 0.17 AU (24 million km) with Messenger S/C (2-way link) and Mars Global Surveyor S/C (1-way link)
- Nd:YAG-Laser, pulse frequency 8 Hz
- Atmospheric correction; calibration via ranging to near-earth satellites (e.g. LAGEOS) from different stations

<table>
<thead>
<tr>
<th>Messenger S/C</th>
<th>MOLA on Mars Global Surveyor (1-way), Separation (2-way)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance</td>
<td>2.4·10⁷ km, 8·10⁷ km</td>
</tr>
<tr>
<td>Pulse-width</td>
<td>10 ns (up), 6 ns (down), 5 ns</td>
</tr>
<tr>
<td>Pulse-energy</td>
<td>16 mJ (up), 20 mJ (down), 150 mJ</td>
</tr>
<tr>
<td>Repetition rate</td>
<td>240 Hz (up), 8 Hz (down), 56 Hz</td>
</tr>
<tr>
<td>Laser energy</td>
<td>3.84 W (up), 0.16 W (down), 8.4 W</td>
</tr>
<tr>
<td>Beam divergence</td>
<td>60 µrad (up), 100 µrad (down), 50 µrad</td>
</tr>
<tr>
<td>Mirror area</td>
<td>0.042 m² (up), 1.003 m² (down), 0.196 m²</td>
</tr>
</tbody>
</table>

John J. Degnan, in Lasers, Clocks, and Drag Free...
LISA: gravitational waves obs.

- Cluster of 3 S/C in heliocentric orbits at 1 AU
- S/C form an equilateral triangle with 5 mio km arm length
- Earth trailing orbit: 20° behind the Earth
- Leaned 60° with respect to the ecliptic
- S/C contain laser and inertial test masses
- System forms a Michelson Interferometer
- Designed for galactic and cosmological sources

K. Danzmann, MPI-AEI
Unexplained phenomena within GR

**Cosmological phenomena**

*Dark Energy (Turner 1999):*  
To describe the accelerated expansion of the universe seen from supernovae observations and CMB anisotropy measurements

*Dark Matter (Zwicky 1933):*  
To describe galactic rotation curves, gravitational lensing effects and early structure formation in cosmological models
Unexplained phenomena within GR

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Astronomical observations

Increase of the Astronomical Unit (Pitjeva 2005, Krasinski 2005):
- length scale related to the earth-sun distance increases by $7 \pm 1$ m per 100 years (confirmed by astronomical observations);
- solar mass loss only explains ca. 1 m per century

Quadrupole/Octopole Anomaly (Tegmark et al. 2005, Schwarz et al. 2005):
- Quadrupole and octopole of CMB are correlated with solar system ecliptic
Increase of the Astronomical Unit

Observations

*Krasinsky and Blumberg* (2005): 15 ± 4 m / 100 a

*Pitjeva* (in *Standish* (2005)): 7± 1 m / 100 a

Remarks and questions

- \( \frac{dG}{dt} \neq 0 \) excluded by Lunar Laser Ranging
- Mass loss of Sun causes only 1 m / 100 a
- Influence by cosmic expansion many orders of magnitude too small
- Increase of solar wind plasma on long time scales?
- Drift of clocks \( t \to t + \alpha t^2 \) with \( \alpha \approx 3 \cdot 10^{-20} \) s\(^{-1}\)?
Observations

- Anomalous behaviour of low $\ell$ contributions to CMB quadupole and octopole aligned to $> 99.87\%$
- Quadrupole and octopole aligned to ecliptic to $> 99\%$
- No correlation with the galactic plane (Oliveira et al. (2004), Schwarz et al. (2005))

Remarks and questions

- Influence of solar system on CMB observations?
- Systematics?
Unexplained phenomena within GR

**Cosmological phenomena**
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**Satellite tracking effects**
- **Pioneer Anomaly** *(Anderson et al. 1998, 2002/04)*
- **GRACE-Anomaly** *(Bertiger et al 2003)*
(1) Pioneer Anomaly

- Acceleration has been observed as constant for more than 16 years
- Temporal and spatial variations < 3%
- Analyzed in detail with data from 1987 to 1998 for distances between 20 and 70 AU by JPL
- Effect observed when satellites had set from elliptic (bound) to hyperbolic (escape) orbits
- No indication about range in space

- Doppler tracking of the Pioneer 10 and 11 satellites showed a blue-shifted, anomalous frequency shift

\[ f_p = (5.99 \pm 0.01) \cdot 10^{-9} \text{ Hz} / \text{s} \]

- Drift can be interpreted as an acceleration directed toward the sun of

\[ a_p = (8.74 \pm 1.33) \cdot 10^{-10} \text{ m/s}^2 \]

(Anderson et al. 1998, 2002)
Reasons for Speculations

- Surprising coincidence of PA acceleration and cosmic expansion rate
  \[ a_p \approx cH \]
- Extremely constant acceleration in space and time
  Canelns out nearly all systematics
- Largest size experiment ever carried out
  Failed to proof Newton’s \(1/r^2\)-law on large distances
- Not the only anomaly observed in our solar system

Doppler Tracking in the expanding Universe

- Observer at rest in cosmic substrate
- S/C moves on geodesics and is slowed down
- Cosmic redshift of frequency
- Resulting Doppler effect (velocity of points of constant distance wrt cosmic substrate)

\[
\nu_2(t_2) = (1 - H(t_2 - t_1) - V_2^{\text{tot}})\nu_0(t_1)
\]

\[
V_2^{\text{tot}}(t_2) = H(t_2 - t_1) - V_2^{\text{tot}}H(t_2 - t_1)
\]

Red shift and Doppler cancel
- Only the satellite’s slow down ist left over.

\[
a = HV = \frac{V}{c} cH \ll cH
\]
## Pioneer 10 and 11 satellites

<table>
<thead>
<tr>
<th></th>
<th>Pioneer 10</th>
<th>Pioneer 11</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Launch</strong></td>
<td>2.3.1972</td>
<td>5.4.1973</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Saturn: 1.9.1979</td>
</tr>
<tr>
<td><strong>Last data received</strong></td>
<td>27.4.2002 (after 30 years of operation)  @ 80.2 AU</td>
<td>1.10.1990 @ ca. 30 AU</td>
</tr>
<tr>
<td><strong>Direction of orbit line</strong></td>
<td>Aldebaran</td>
<td>Aquila constellation</td>
</tr>
<tr>
<td><strong>Satellite mass</strong></td>
<td>259 kg</td>
<td></td>
</tr>
<tr>
<td><strong>Power: SNAP-19 RTGs</strong></td>
<td>boom 3 m / mass 13.6 kg</td>
<td></td>
</tr>
<tr>
<td><strong>Magnetometer</strong></td>
<td>boom 6 m / mass 5 kg</td>
<td></td>
</tr>
<tr>
<td>**High-gain antenna Ø</td>
<td>2.74 m</td>
<td></td>
</tr>
<tr>
<td><strong>Maximum cross section</strong></td>
<td>5.914 m²</td>
<td></td>
</tr>
<tr>
<td><strong>Spin rate</strong></td>
<td>4.28 rpm</td>
<td></td>
</tr>
<tr>
<td><strong>Maximum moment of inertia</strong></td>
<td>588.3 kg · m²</td>
<td></td>
</tr>
<tr>
<td><strong>Up-link frequency</strong></td>
<td>2.110 GHz (S-band)</td>
<td></td>
</tr>
<tr>
<td><strong>Down-link frequency</strong></td>
<td>2.292 GHz (S-band)</td>
<td></td>
</tr>
<tr>
<td><strong>Radio link wavelength</strong></td>
<td>ca. 13 cm</td>
<td></td>
</tr>
<tr>
<td><strong>Transmission power</strong></td>
<td>8 W</td>
<td></td>
</tr>
</tbody>
</table>
The orbits of Pioneer 10 and 11

- Elliptic (bound) orbits before the last fly-by
- Hyperbolic (escape) orbits after the last fly-by
Observed Anomalous Doppler Drift

frequency received at S/C:
\[ f' = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \left( 1 - \frac{v}{c} \right) \cdot f \]

frequency sent back and received on earth:
(neglecting the transponder shift)
\[ f'' = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \left( 1 - \frac{v}{c} \right) \cdot f' \]

\[
\frac{f'' - f}{f} = -2 \frac{v}{c} \cdot \frac{v}{1 + v/c} \approx -2 \frac{v}{c}
\]

\[ V_{\text{observed}} - V_{\text{modelled}} = -2a_p t \]

The two-way Doppler residuals for Pioneer 10 vs time
[1 Hz is equal to 65 mm/s range change per second].
## Error budget of external effects

<table>
<thead>
<tr>
<th>Error budget constituents</th>
<th>Bias [10^{-10}m/s^2]</th>
<th>Uncertainty [10^{-10} m/s^2]</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sources of external systematics</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Solar radiation pressure</td>
<td></td>
<td>± 0.001</td>
</tr>
<tr>
<td>→ Sol. rad. press. from mass uncertainties</td>
<td>+ 0.03</td>
<td>± 0.01</td>
</tr>
<tr>
<td>Solar wind</td>
<td></td>
<td>± 0.00001</td>
</tr>
<tr>
<td>Solar corona effects</td>
<td></td>
<td>± 0.02</td>
</tr>
<tr>
<td>Lorentz force (em-effects)</td>
<td></td>
<td>± 0.0001</td>
</tr>
<tr>
<td>Kuiper belt's gravity</td>
<td></td>
<td>± 0.03</td>
</tr>
<tr>
<td>Earth rotation</td>
<td></td>
<td>± 0.001</td>
</tr>
<tr>
<td>Mechanical / phase stability of DSN antenna</td>
<td></td>
<td>± 0.001</td>
</tr>
<tr>
<td>Clock effects on phase stability</td>
<td></td>
<td>± 0.001</td>
</tr>
<tr>
<td>DSN station location</td>
<td></td>
<td>± 0.00001</td>
</tr>
<tr>
<td>Tropospheric and ionospheric effects</td>
<td></td>
<td>± 0.001</td>
</tr>
<tr>
<td><strong>Computational systematics</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Numerical stability of least-square estimation</td>
<td></td>
<td>± 0.02</td>
</tr>
<tr>
<td>Accuracy of consistency / model tests</td>
<td></td>
<td>± 0.13</td>
</tr>
<tr>
<td>→ Mismodelling of manoeuvres</td>
<td></td>
<td>± 0.01</td>
</tr>
<tr>
<td>→ Mismodelling of solar corona</td>
<td></td>
<td>± 0.02</td>
</tr>
<tr>
<td>Annual / diurnal terms</td>
<td></td>
<td>± 0.32</td>
</tr>
</tbody>
</table>
# Sources of internal systematic error

<table>
<thead>
<tr>
<th>Error Budget Constituents</th>
<th>Bias [10^{-10} m/s^2]</th>
<th>Uncertainty [10^{-10} m/s^2]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radio beam reaction force</td>
<td>+1.10</td>
<td>±0.11</td>
</tr>
<tr>
<td>Thermal and propulsion effects from RTGs</td>
<td>-0.55</td>
<td>±0.55</td>
</tr>
<tr>
<td>→ RTG heat reflected off the S/C</td>
<td>-0.55</td>
<td>±0.55</td>
</tr>
<tr>
<td>→ Differential emissivity of the RTGs</td>
<td>±0.85</td>
<td></td>
</tr>
<tr>
<td>→ Non-isotropic radiative cooling of S/C</td>
<td>±0.16</td>
<td></td>
</tr>
<tr>
<td>→ Expelled He produced within the RTGs</td>
<td>+0.15</td>
<td>±0.16</td>
</tr>
<tr>
<td>Mass expulsion / gas leakage</td>
<td>±0.56</td>
<td></td>
</tr>
<tr>
<td>Variation between S/C determinations</td>
<td>+0.17</td>
<td>±0.17</td>
</tr>
</tbody>
</table>

\[ \Delta V = S \Delta T \]

*S - Seebeck coefficient*

---

**Radioisotope Thermoelectrical Generator (SNAP-19)**

\[ ^{94}\text{Pu}^{238} \rightarrow ^{92}\text{U}^{234} + ^2\text{He}^4 \]

Half-life time: 87.74 years
ca. 2000 W
Thermal history

- 60 W asymmetric IR-radiation (out of initial 2000 W) could explain the anomaly
- Symmetry breaking through high-gain antenna (Ø 2.3 m)
- Spin rate change of both satellites shows, that thermal models are consistent.
- Models include surface degradation effects (also worst-case scenarios)
Unexpected masses in the solar system

Acceleration vs. distance for different mass density distribution (Nieto, 2005)

ring with mass density $\mu(\rho) \sim 1/\rho$

needs ca. 40 Earth masses
Drag through dust

Interplanetary Medium

- is a thinly scattered matter (neutral Hydrogen, microscopic particles) with two main contributions, IPD and ISD:

- Interplanetary Dust (IPD), modelled: \( \rho_{IPD} \leq 10^{-24} \text{ g/cm}^3 \)

- Interstellar Dust (ISD), measured on Ulysses S/C: \( \rho_{ISD} \leq 10^{-26} \text{ g/cm}^3 \)

Drag on a spacecraft

\[
a_{drag} = -c_s \rho(r) \nu_s A_s \frac{A_s}{m_s}
\]

The Pioneer Anomaly (between 20 and 70 AU) could only be explained with an axially-symmetric dust distribution with a constant uniform density of

\[
\rho(r) \leq \rho_0 = 3 \cdot 10^{-19} \text{ g/cm}^3 \approx 300.000(\rho_{IPD} + \rho_{ISD})
\]
Yukawa modification

Ansatz \[ V(r) = G \frac{M_{\text{Sun}}}{r} \left( 1 + \alpha \cdot e^{-r/\lambda} \right) \]

with Taylor extension and \( G_0 = (1 + \alpha)G \) as observed grav. constant for \( r \to \infty \)

\[ a(r) = -G_0 \frac{M_{\text{Sun}}}{r^2} + \frac{\alpha}{1 + \alpha} G_0 \frac{M_{\text{Sun}}}{2 \lambda^2} - \frac{\alpha}{1 + \alpha} G_0 \frac{M_{\text{Sun}}}{3 \lambda^2} \frac{r}{\lambda} + \ldots \]

- \( \log_{10}(\lambda) > 16 \) for \( \log_{10}|\alpha| = 1 \) compatible with present experimental results in the solar system (including planetary orbits)

- A viable model?
  - Pioneer Anomaly: \( \log_{10}(\lambda) > 16, \alpha + 1 \leq 10^{-5} \)
  - Galaxy rotation curves \( \log_{10}(\lambda) > 16, \alpha + 1 \leq 10^{-1} \)
(2) Fly-by Anomaly

1st time measured Dec. 1990 during GALILIEO’s 1st gravity assist manoeuvre at Earth:

unexplained 2-way S-band Doppler shift of 66 mHz which could be interpreted as anomalous velocity increase of $3.93 \pm 0.08 \text{ mm/s}$

# Earth Fly-bys analyzed so far

<table>
<thead>
<tr>
<th></th>
<th>Galileo (1\textsuperscript{st} fly-by)</th>
<th>NEAR</th>
<th>Cassini</th>
<th>Rosetta</th>
<th>Messenger</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_\infty$ [km/s]</td>
<td>8.949</td>
<td>6.851</td>
<td>16.01</td>
<td>3.863</td>
<td>4.056</td>
</tr>
<tr>
<td>$h$ [km]</td>
<td>956</td>
<td>532</td>
<td>1,172</td>
<td>1,954</td>
<td>2,336</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>2.47</td>
<td>1.81</td>
<td>5.86</td>
<td>1.31</td>
<td>1.13</td>
</tr>
<tr>
<td>$\Theta$ [°]</td>
<td>47.67</td>
<td>66.92</td>
<td>19.66</td>
<td>99.396</td>
<td>94.7</td>
</tr>
<tr>
<td>$i$ [°]</td>
<td>142.9</td>
<td>108.0</td>
<td>25.4</td>
<td>144.9</td>
<td>133.1</td>
</tr>
<tr>
<td>$\Delta V_\infty$ [mm/s]</td>
<td>3.92 $\pm$ 0.08</td>
<td>13.46 $\pm$ 0.13</td>
<td>$\sim$ 1</td>
<td>1.82 $\pm$ 0.05</td>
<td></td>
</tr>
<tr>
<td>$\Delta V_F$ [mm/s]</td>
<td>2.56 $\pm$ 0.05</td>
<td>7.24 $\pm$ 0.07</td>
<td>[-0.2] (?)</td>
<td>0.67 $\pm$ 0.02</td>
<td>O (0)</td>
</tr>
</tbody>
</table>
Observed phenomena

$\Delta E_{\text{kin}} / m_{\text{sat}} = \left( v_0^2 - v_i^2 \right) / 2$

$= v_{\text{earth}} \left( v_0' - v_i' \right)$

- $\Delta v$ decreases with increasing eccentricity and perigee height
- $\Delta v$ disappears at $e = 1$
  (as expected for bound orbits)
## Error analysis

<table>
<thead>
<tr>
<th>Error budget constituents</th>
<th>Bias [10^-5 m/s^2]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atmospheric drag</td>
<td>-0.0001</td>
</tr>
<tr>
<td>Ocean tides</td>
<td>±0.1</td>
</tr>
<tr>
<td>Solid earth tides</td>
<td>≪</td>
</tr>
<tr>
<td>S/C charging (modeled / analyzed for LISA; for charging Q &lt; 10^-7 C)</td>
<td>±0.0001</td>
</tr>
<tr>
<td>Magnetic moments (&lt; 2 · 10^-7 G/m)</td>
<td>±10^-10</td>
</tr>
<tr>
<td>Earth albedo (1 t S/C)</td>
<td>±0.00024</td>
</tr>
<tr>
<td>Solar wind</td>
<td>±0.0003</td>
</tr>
<tr>
<td>Relativistic corrections V · v^2 / c^2 ≈ 10^-20</td>
<td>not affecting</td>
</tr>
<tr>
<td>Spin rotation coupling (coupling of the helicity of radio waves with S/C spin and Earth rotation (only effective for 2-way Doppler ranging))</td>
<td>not affecting</td>
</tr>
</tbody>
</table>
Attempts to explain

- Explanations by systematics failed so far.
- Confirmed by different codes
- Real effect inherent to the tracking of S/C
- Source unknown

(Anderson et al., PRL, 2008)

Empirical prediction formula (Anderson et al., PRL, 2008)

\[ V_\infty^2 = \bar{V}_{S/C} \cdot \bar{V}_{S/C} - \frac{2M_E G}{r} \]

\[ \frac{\Delta V_\infty}{V_\infty} = K \left( \cos \delta_{in} - \cos \delta_{out} \right) \quad \text{with} \quad K = \frac{2\omega_E R_E}{c} \]
(3) GRACE Anomaly

- Observed difference of a systematic time shifts of $0.056 \text{ ps/s} \rightarrow 45.6 \text{ ns/d}$ independently determined by
  (1) Ku-band ranging and
  (2) GPS (Bertiger et al. 2003)

- Could be interpreted as a anomalous acceleration of the GRACE satellites:
  $0.2 \cdot 10^{-4} \text{ m/s}^2$

- Same order of magnitude than fly-by anomaly!
Take-home messages

• Unexplained phenomena
  • Dark matter (does it affect solar system physics?)
  • Dark energy
  • Increase of AU
  • Quadrupole / Octopole anomaly
  • Pioneer Anomaly
  • It´s worth to discuss the anomalies (not understood so far)
  • Try to find systematics
  • Try to find conventional explanations
  • Try to find relations between anomalies (Anomalies most probably are not isolated phenomena.)
  • Are there similar effects in other gravitating systems?
  • What´s about hyperbolic orbits?
• Observation of future fly-bys of satellites
  • Rosetta Earth fly-by 11 / 2009 (orbital height: ca. 2,500 km)
  • New Horizon Jupiter fly-by in 2008?