Minicharges and Magnetic Monopoles

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(with Jörg Jäckel and Valya Khoze)

The most general* $\mathrm{U}(1) \times \mathrm{U}(1)'$ Lagrangian

$$\mathcal{L} = -\frac{1}{4}\left(F_{\mu\nu}F^{\mu\nu} + G_{\mu\nu}G^{\mu\nu}\right)$$

*4d, renormalizable, gauge sector only, etc.

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$$\mathcal{L} = -\frac{1}{4}\left(F_{\mu\nu}F^{\mu\nu} + G_{\mu\nu}G^{\mu\nu} + 2\chi\, F_{\mu\nu}G^{\mu\nu}\right)$$

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$$-\frac{1}{32\pi^2}\left(\theta_F\, F_{\mu\nu}\tilde{F}^{\mu\nu} + \theta_G\, G_{\mu\nu}\tilde{G}^{\mu\nu} + 2\theta_{FG}\, F_{\mu\nu}\tilde{G}^{\mu\nu}\right)$$

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- Nonzero χ, kinetic mixing:
  Important for charged particles (Minicharges)
- Nonzero θs:
  Important for magnetic monopoles (Witten effect, Magnetic Mixing)

In this talk:

- Can we have both minicharges and magnetic monopoles?
  What about Dirac quantization?
- Can there be observable consequences of nonzero θs?
  $\rightarrow$ Magnetic Mixing!

*4d, renormalizable, gauge sector only, etc.

Outline

1. Kinetic mixing and minicharges
2. Minicharges and monopoles
3. Magnetic mixing
4. Conclusions

Kinetic mixing and minicharges

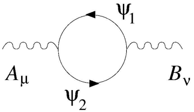
Reminder: Minicharges

θs $= 0$ for now.

$\mathrm{U}(1) \times \mathrm{U}(1)'$ gauge theory, field strengths $F = dA$ and $G = dB$:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} - \frac{\chi}{2}F^{\mu\nu}G_{\mu\nu} - ej_\mu A^\mu - e'j'_\mu B^\mu$$

- Kinetic mixing term allowed by symmetries
- Even if $\chi = 0$ at tree level, $\chi \neq 0$ can be radiatively generated (if there are massive fields ψ_i charged under both $\mathrm{U}(1)$ factors)



(→ Holdom '86)

- χ is arbitrary, irrational: $\chi \approx \frac{e\,e'}{6\pi^2} \log \frac{m_1}{m_2}$

Reminder: Minicharges

Diagonalize $\mathrm{U}(1)$ gauge fields:

$$C \equiv B + \chi A, \qquad H \equiv dC = G + \chi F$$

$$\Rightarrow \qquad \mathcal{L} = -\frac{1}{4}(1-\chi^2)F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}H_{\mu\nu}H^{\mu\nu} - ej_\mu A^\mu - e'j'_\mu C^\mu + \chi e' \, j'_\mu A^\mu$$

Now current j' couples to gauge field A with charge $-\chi e'$.

Usual picture:

- A, F = photon and field strength of electromagnetism
- B, G = gauge field and field strength of hidden sector
- Hidden matter fields pick up EM charges $\sim \chi$: minicharged particles
- χ must be tiny to evade experimental constraints
  ($\rightarrow$ many talks at this conference…)

Minicharges and monopoles

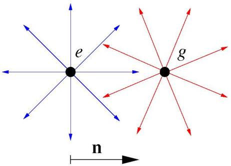
Another reminder: The Dirac Monopole

Dirac quantization condition for $\mathrm{U}(1)$ magnetic monopoles:

$$\frac{eg}{2\pi} \in \mathbb{Z}$$

g = magnetic, e = electric charge

One derivation (out of many): Static electron-monopole configuration



carries angular momentum:

$$\mathbf{L} = \int d^3x\, \mathbf{x} \times (\mathbf{E} \times \mathbf{B}) = \frac{eg}{4\pi}\,\mathbf{n}, \qquad |\mathbf{L}| = \frac{eg}{4\pi}$$

Angular momentum quantization $\Rightarrow$ Dirac quantization condition

Monopoles and electric charge quantization

If $eg/2\pi \in \mathbb{Z}$ for all electric charges e, then $e_1/e_2 \in \mathbb{Q}$ for any two charges $e_{1,2}$.

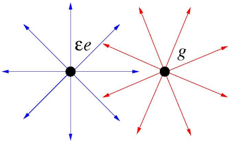
A puzzle: How is this compatible with minicharged particles?

Kinetic mixing generated radiatively $\Rightarrow$ minicharges arbitrary, irrational

Example: For

- electron with electric charge e
- magnetic monopole with magnetic charge $g = 2\pi n/e$
- minicharged particle with electric charge $-\chi e' \equiv \epsilon e$

the static configuration



has angular momentum $|\mathbf{L}| = n\epsilon/2 \notin \frac{1}{2}\mathbb{Z}$!
This type of monopole cannot be consistently added to the theory.

A generalized Dirac quantization condition

Solution: Permit only monopoles with suitable magnetic charges also under the hidden $\mathrm{U}(1)$.

Particle	q_{vis}	q_{hid}	g_{vis}	g_{hid}
electron	e	0	0	0
minicharged particle	$-\chi e'$	e'	0	0
monopole	0	0	g	g'

We have seen that $g' = 0$ is not allowed.
But: With

- $(g, g') = \left(0, \frac{2\pi n}{e'}\right)$
- or $(g, g') = \left(\frac{2\pi m}{e}, \frac{2\pi\chi m}{e}\right)$

$(m, n \in \mathbb{Z})$

the total field angular momentum of MCP-monopole system

$$\mathbf{L} = \int d^3x\, \mathbf{x} \times (\mathbf{E}_{\mathrm{vis}} \times \mathbf{B}_{\mathrm{vis}} + \mathbf{E}_{\mathrm{hid}} \times \mathbf{B}_{\mathrm{hid}}) = \frac{eg - \chi e' g}{4\pi} + \frac{e' g'}{4\pi}$$

is half-integral for all configurations.

Minicharges and magnetic monopoles

In terms of original basis (F and G field strengths):
Field angular momentum gets contribution from kinetic mixing term,

$$\mathbf{L} = \int d^3x\, \mathbf{x} \times \left(\mathbf{E}_{\text{vis}} \times \mathbf{B}_{\text{vis}} + \chi \left(\mathbf{E}_{\text{hid}} \times \mathbf{B}_{\text{vis}} + \mathbf{E}_{\text{vis}} \times \mathbf{B}_{\text{hid}}\right) + \mathbf{E}_{\text{hid}} \times \mathbf{B}_{\text{hid}}\right)$$

In that basis, allowed monopoles have F- and G-magnetic charges

- $(g, g') = \left(\frac{2\pi n}{e}, 0\right)$
- or $(g, g') = \left(0, \frac{2\pi m}{e'}\right)$

corresponding to the F- and H-magnetic charges found before.

The 't Hooft-Polyakov monopole and kinetic mixing

Some models necessarily contain monopoles with calculable properties.
Example: $\mathrm{SU}(2)$ gauge theory, broken to $\mathrm{U}(1)$ by adjoint Higgs:

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu\, a} - \frac{1}{2} (D_\mu \phi)^a (D^\mu \phi)^a + m^2 \phi^a \phi^a - \lambda (\phi^a \phi^a)^2$$

"Hedgehog vacuum", $\langle \phi^a \rangle = r^a f(r)$: breaks $SU(2) \rightarrow \mathrm{U}(1)$ at large r, magnetic monopole at $r = 0$ ($\rightarrow$ 't Hooft, Polyakov '74)

Add an extra $\mathrm{U}(1)$ with field strength F:

$$\Delta\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2M} \phi^a G^{\mu\nu\, a} F_{\mu\nu}$$

- Last term $\Leftarrow$ integrated out heavy states charged under F and G
- Gives kinetic mixing between unbroken $\mathrm{U}(1)' \subset \mathrm{SU}(2)$ and extra $\mathrm{U}(1)$:

$$\chi = \frac{|\langle \phi \rangle|}{M}$$

- Can show: Monopole satisfies generalized Dirac quantization condition

Several $\mathrm{U}(1)$s and dyons

N $\mathrm{U}(1)$ factors labeled by I, J:

$$\mathcal{L} = -\frac{1}{4} f_{IJ} F^{I\,\mu\nu} F^{J}_{\mu\nu} - e^{\alpha}_{I} A^{I}_{\mu} j^{\mu}_{\alpha},$$

Static electric point sources labeled by α:

$$(j^{\mu}_{\alpha}) = \left(\delta^3(\mathbf{x} - \mathbf{x}_\alpha),\, \mathbf{0}\right)$$

Also allow for monopole charges $g^{\alpha\,I}$: “dyons”

Angular momentum of a two-dyon system is

$$4\pi\,\mathbf{L} = \int d^3x\,\mathbf{x} \times \left(\mathbf{E}^{1\,I} \times \mathbf{B}^{2\,J} - \mathbf{E}^{2\,I} \times \mathbf{B}^{1\,J}\right) f_{IJ} = e^1_J g^{2\,J} - e^2_J g^{1\,J}$$

Gives generalized Dirac-Zwanziger-Schwinger charge quantization condition:

$$e^{\alpha}_{I} g^{\beta\,I} - e^{\beta}_{I} g^{\alpha\,I} \in 2\pi\mathbb{Z} \qquad \forall\,(\alpha, \beta)$$

which is invariant under basis changes (with regular matrix $M = (M^I_J)$)

$$A \to M^{-1} A, \quad F \to M^{-1} F, \quad f \to M^T f\, M, \quad e \to M^T e, \quad g \to M^{-1} g$$

Magnetic mixing

Yet another reminder: The Witten effect

($\to$ Witten '79)

A θ-term as in

$$\mathcal{L} = -\frac{1}{4e^2} F^{\mu\nu} F_{\mu\nu} - \frac{\theta}{32\pi^2} F^{\mu\nu} \tilde{F}_{\mu\nu}$$

gives electric charges to magnetic monopoles:

For magnetic monopole background (charge g) with superimposed static potential $(A^0, \mathbf{A})$,

$$-\frac{\theta}{32\pi^2} F^{\mu\nu} \tilde{F}_{\mu\nu} = \frac{\theta}{8\pi^2} \mathbf{E} \cdot \mathbf{B} = \frac{\theta}{8\pi^2} \left(\nabla A^0\right) \cdot \left(\nabla \times \mathbf{A} + \frac{ge}{4\pi} \frac{\mathbf{r}}{r^3}\right)$$

$$L = \int d^3 r \, \mathcal{L} \supset -\frac{\theta}{8\pi^2} \int d^3 r \, A^0 \, \nabla \cdot \frac{ge \, \mathbf{r}}{4\pi \, r^3} = -\frac{\theta eg}{8\pi^2} \int d^3 r \, A^0 \delta^3(\mathbf{r})$$

RHS = electric point charge $-\theta eg/(8\pi^2)$, located at at $\mathbf{r} = 0$, couples to electrostatic potential A^0

The monopole has picked up an electric charge $\sim \theta$.

Magnetic mixing

Our most general* $\mathrm{U}(1) \times \mathrm{U}(1)'$ Lagrangian had an off-diagonal θ-term:

$$\mathcal{L} \supset -\frac{\theta_{FG}}{16\pi^2} F_{\mu\nu} \tilde{G}^{\mu\nu}$$

A G-monopole will pick up an F-electric charge $\sim \theta_{FG}$
If F is electromagnetism, and G is a hidden sector field:

Hidden sector monopoles pick up ordinary electric minicharges.

Abelian θ-terms are usually ignored because visible sector monopoles are superheavy, $M \gg M_{\mathrm{GUT}}$.

But hidden sector monopoles could be light if breaking scale is low or hidden sector is strongly coupled.

A basis-independent and duality invariant observable

- Have seen: Notion of "charge" is basis-dependent in presence of kinetic/magnetic mixing
- In addition: For electromagnetism coupled to massive point sources, electric/magnetic degrees of freedom interchangeable (E-M duality)

Observable quantities should be basis-independent and duality invariant

Example: Potential energy between two static point sources, with electric (magnetic) monopole number vectors n^e (n^m)

Notation:

$$\mathcal{L} = \frac{1}{16\pi} \int d^4x \, \left[(\tau_1)_{IJ} F^I_{\mu\nu} \tilde{F}^{J\,\mu\nu} + (\tau_2)_{IJ} F^I_{\mu\nu} F^{J\,\mu\nu} \right]$$

where

$$\tau_{IJ} = (\tau_1)_{IJ} + \mathrm{i}(\tau_2)_{IJ} = \left(\frac{\theta}{2\pi}\right)_{IJ} + \mathrm{i}\left(\frac{4\pi}{e^2}\right)_{IJ}$$

Potential energy is then

$$\mathcal{E} = \frac{1}{r} \left(n^{e\prime T} n^{m\prime T} \right) \begin{pmatrix} \tau_2^{-1} & -\tau_2^{-1}\tau_1 \\ -\tau_1\tau_2^{-1} & \tau_1\tau_2^{-1}\tau_1 + \tau_2 \end{pmatrix} \begin{pmatrix} n^e \\ n^m \end{pmatrix}$$

A basis-independent and duality invariant observable

Potential energy between two static dyons:

$$\mathcal{E} = \frac{1}{r}\,({n^{e\prime}}^T {n^{m\prime}}^T)\begin{pmatrix} \tau_2^{-1} & -\tau_2^{-1}\tau_1 \\ -\tau_1\tau_2^{-1} & \tau_1\tau_2^{-1}\tau_1 + \tau_2 \end{pmatrix}\begin{pmatrix} n^e \\ n^m \end{pmatrix}$$

- invariant under change of basis in $\mathrm{U}(1)^N$ space of gauge fields
- invariant under $\mathrm{Sp}(2N,\mathbb{Z})$ duality
  (generalizing $\mathrm{SL}(2,\mathbb{Z})$ electric-magnetic duality to several $\mathrm{U}(1)$ factors)

Here $\begin{pmatrix} A & B \\ C & D \end{pmatrix} \in \mathrm{Sp}(2N,\mathbb{Z})$ acts as

$$\tau \mapsto (A\tau + B)(C\tau + D)^{-1}, \quad \begin{pmatrix} n^e \\ n^m \end{pmatrix} \mapsto \begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} n^e \\ n^m \end{pmatrix}$$

(A, B, C, D are $N \times N$ matrices)

Radiatively generating magnetic mixing

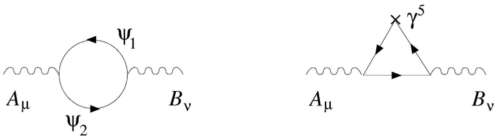
Example with gauge group $\mathrm{U}(1) \times \mathrm{SU}(2)$, adjoint Higgs Φ
Add Dirac fermion Ψ (charged $\mathrm{SU}(2)$ doublet) with CP-violating masses and Yukawa couplings:

$$\mathcal{L} = \mathcal{L}_{\text{gauge}} - (D\Phi)^2 - V(\Phi) + i\bar{\Psi}\not{D}\Psi - m_1\bar{\Psi}\Psi - \mathrm{i}m_2\bar{\Psi}\gamma^5\Psi - h_1\Phi\bar{\Psi}\Psi - \mathrm{i}h_2\Phi\bar{\Psi}\gamma^5\Psi$$

After breaking to $\mathrm{U}(1) \times \mathrm{U}(1)'$: Doublet components of Ψ get mass matrix

$$M = \begin{pmatrix} m_1 + \mathrm{i}m_2\gamma^5 & 0 \\ 0 & m_1 + \mathrm{i}m_2\gamma^5 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} h_1 v + \mathrm{i}h_2 v\gamma^5 & 0 \\ 0 & -h_1 v - \mathrm{i}h_2\gamma^5 \end{pmatrix}$$

Kinetic and magnetic mixing induced by mass splittings via



- non-zero h_1 $\rightarrow$ kinetic mixing
- non-zero h_2 $\rightarrow$ magnetic mixing

Non-abelian embedding

$\mathrm{U}(1)^k$ structure can come from single non-abelian gauge group

- Gauge group Higgsed to $\mathrm{U}(1)^k$.
- Monopoles = ’t Hooft-Polyakov monopoles
- Kinetic mixing generated by charged matter
- Magnetic mixing generated by charged matter with CP-violating couplings

Toy example: Seiberg–Witten theory with matter (→ Seiberg, Witten ’94)

- $\mathrm{SU}(N)$ gauge theory with $\mathcal{N} = 2$ SUSY
- adjoint Higgs from $\mathcal{N} = 2$ gauge multiplet breaks $\mathrm{SU}(N) \rightarrow \mathrm{U}(1)^N$ at generic point in moduli space
- gauge couplings and θ angles can be explicitly calculated (one-loop exact, non-perturbative corrections known)
- at strong coupling, monopoles become light

May speculate that this could happen in a realistic strongly coupled hidden sector — could detect hidden monopoles by their minicharges!

Conclusions

- Models with several $\mathrm{U}(1)$ factors, or non-abelian gauge groups Higgsed to $\mathrm{U}(1)^k$, naturally lead to minicharged particles
- They may still allow for magnetic monopoles if the Dirac quantization condition is suitably generalized
- If (one of) the $\mathrm{U}(1)$ factors comes from a non-abelian gauge group, and the monopole is a 't Hooft-Polyakov monopole, then the modified Dirac quantization condition is automatically satisfied as expected
- Ordinary θ-terms $\Rightarrow$ electric charges for magnetic monopoles: Witten effect
- Off-diagonal θ-terms $\Rightarrow$ visible electric charges for hidden sector monopoles: magnetic mixing
- θ-terms induced by CP-violating hidden matter fields
- A hidden sector monopole may be light (and thus detectable), e.g. if it is the 't Hooft-Polyakov monopole of a non-abelian gauge group
  - with a low breaking scale
  - or at strong coupling