Photon Production From The Scattering of Axions From a Solenoidal Magnetic Field

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with

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Outline

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Introduction
O(2) Symmetry

- The axion-photon system is described by the action

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m_a^2\phi^2 - -\frac{g}{8}\phi\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}$$

- Considering a strong external magnetic field in the z direction

$$\mathcal{L}_I = -gB(x,y)\phi E_z = -\beta\phi E_z$$

- Only the z-polarization of the EM wave couples to the axion. Hence, by considering only this polarization and choosing the temporal gauge for the EM field we get the effective Lagrangian

$$\mathcal{L}_2 = \frac{1}{2}\partial_\mu A\partial^\mu A + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m_a^2\phi^2 + \beta\phi\partial_t A$$

- Neglecting the mass of the axion, gives an O(2) symmetry in the axion-photon field space (by the axion and photon kinetic terms). We may also express the interaction term in an O(2) symmetric way

$$\mathcal{L}_I = \frac{1}{2}\beta(\phi\partial_t A - A\partial_t\phi)$$

Introduction

Complex Axion-Photon Field

- Now, In the infinitesimal limit there is an axion-photon O(2) symmetry (ordinary rotation in the axion-photon space), which allows us to define a complex scalar field

$$\Psi = \frac{1}{\sqrt{2}}(\phi + iA)$$

- The Lagrangian for the complex field is

$$\mathcal{L} = \partial_\mu \Psi^* \partial^\mu \Psi - \frac{i}{2}\beta(\Psi^* \partial_t \Psi - \Psi \partial_t \Psi^*)$$

- Hence the EOM for the complex field is

$$\partial_\mu \partial^\mu \Psi + i\beta \partial_t \Psi = 0$$

- Introducing the charge conjugation (CC), $\Psi \to \Psi^*$, shows that the free part of the Lagrangian is invariant under this transformation.

Introduction

S and AS parts

- The photon is an AS combination of particle and anti-particle, while an axion is a symmetric combination

$$\phi = \frac{1}{\sqrt{2}}(\Psi^* + \Psi) \quad \text{and} \quad A = \frac{1}{i\sqrt{2}}(\Psi - \Psi^*)$$

- The axion is even under CC, while the photon is odd. These two eigenstates of CC will propagate without mixing as long as no external magnetic field in the perpendicular direction (B) to the eigenstates' spatial dependence is applied.

- The interaction term is not invariant under CC. Thus, the S and AS combinations will not be preserved in the presence of B, since it breaks the symmetry between the particles and anti-particles.

Introduction
Symmetry Breaking

- Particles and anti-particles will suffer opposite forces under the influence of B. Thus, the S and AS combinations will be decomposed through scattering into their particles and anti-particles components.
- The scattering amplitudes of particles and anti-particles have opposite signs: calling the scattering amplitude for a particle S, the amplitude for an anti-particle is then -S.
- Therefore, an axion (1,1) goes under scattering to (1,1) + (S,-S). Hence, the scattering amplitude for an axion to become a photon (1,-1) is S.
- ***The amplitude for axion-photon conversion is equal to the particle scattering amplitude!***

2 Infinitely Thin Solenoid

Delta Function Potential

- Writing separately the time and space dependence of particles as $\Psi = \mathrm{e}^{-i\omega t}\psi(\vec{r})$ and considering a magnetic field of the form $B = \Phi\delta^2(x,y)$ gives the following EOM

$$[-\vec{\nabla}^2 + g\Phi E\delta^2(x,y)]\psi(\vec{r}) = E^2\psi(\vec{r})$$

- In terms of momentum space wave functions we have then

$$\vec{k}^2\phi(\vec{k}) + g\Phi E\psi(0) = E^2\phi(\vec{k}) \Rightarrow \phi(\vec{k}) = (2\pi)^2\delta^2(\vec{k}-\vec{k}_0) - \frac{g\Phi E\psi(0)}{k^2 - E^2}$$

- To obtain the scattering amplitudes, we write the solution back in position space

$$\psi(\vec{r}) = \mathrm{e}^{i\vec{k}\cdot\vec{r}} - g\Phi E\psi(0)G_k(r) \Rightarrow f(\theta) = -\frac{1}{\sqrt{2\pi E}}\frac{g\Phi E}{2}\psi(0)$$

- The delta function approximation gives a constant scattering amplitude.

2 Infinitely Thin Solenoid

Delta Function Potential

$$\sigma_{tot}^{\delta} = \frac{g^2 \Phi^2 E}{4}$$

$$P_\delta = \sigma_S / \sigma_G = \frac{g^2 \Phi^2 E}{4D} = \frac{\pi^2 g^2 B^2 R^3 E}{8}$$

B [Tesla]	D [cm]	σ_{tot}^{δ} [cm]	N_δ [1/sec]	P_δ
10	1	3.08E-15	0.01	3.08E-15
10	10	3.08E-11	112.98	3.08E-12
6	2	1.77E-14	0.07	8.87E-15
6	20	1.77E-10	650.78	8.87E-12

3 Finite Sized Solenoids
Gaussian Distributed Field

- Considering a Gaussian shape magnetic field

$$\vec{B}(r) = B_0 \mathrm{e}^{\frac{-r^2}{R^2}}\,\hat{z}$$

- Using the asymptotic form of Green's function in 2D we arrive at

$$\psi(\vec{r}) = \mathrm{e}^{i\vec{k}\cdot\vec{r}} + \frac{\mathrm{e}^{ikr}}{2\sqrt{2\pi r E}} \int gEB(\vec{\rho})\mathrm{e}^{i\vec{q}\cdot\vec{\rho}} d^2\rho$$

- The Fourier transform of a Gaussian is a Gaussian

$$\psi(\vec{r}) = \mathrm{e}^{i\vec{k}\cdot\vec{r}} + \frac{\sqrt{\pi} g B_0 R^2 \sqrt{E}}{2\sqrt{2r}} \mathrm{e}^{-\frac{1}{4}(Rq)^2} \mathrm{e}^{i(kr+\pi/4)}$$

- And we find the scattering amplitude

$$f(\theta) = \sqrt{(\pi/8)}\, g B_0 R^2 E^{1/2} \mathrm{e}^{-\frac{1}{4}(Rq)^2}$$

- Where the explicit dependence of q on the angle is given by

$$q^2 = 2k^2(1-\cos\theta) = 4k^2 \sin^2(\theta/2)$$

3 Finite Sized Solenoids

Gaussian Distributed Field

$$\sigma_{tot.}^{Gauss} = \int_0^{2\pi} |f(\theta)|^2 d\theta = \frac{\pi^2}{4}(gB_0)^2 R^4 E \mathrm{e}^{-(Rk)^2} I_0((Rk)^2)$$

$$\sim \sigma_{tot}^{Gauss} = \frac{\pi^{3/2}}{\sqrt{32}} g^2 B_0^2 R^3 \qquad P_{Gauss} = \frac{\pi^{3/2}}{8\sqrt{2}} g^2 B_0^2 R^2$$

B [Tesla]	D [cm]	$\sigma_{tot.}^{Gauss}$ [cm]	N_{Gauss} [1/sec]	P_{Gauss}
10	1	1.17E-23	4.29E-11	1.17E-23
10	10	1.17E-20	4.29E-08	1.17E-21
6	2	3.38E-23	1.24E-10	1.69E-23
6	20	3.38E-20	1.24E-07	1.69E-21

3 Finite Sized Solenoids

Ideal Solenoid

- Taking the solenoid as ideal, we have the step function shape for the magnetic field

$$\vec{B}(r) = \begin{cases} B_0 \hat{z} \ , & r < a \ , \\ 0 \ , & r > a \end{cases}$$

- The Fourier Transform of the step function in 2D is the Bessel function

$$f(\theta) = \sqrt{\frac{\pi}{2}} \frac{B_0 a g E^{1/2}}{q} \mathrm{J}_1(qa)$$

- The total 2D cross-section can be evaluated only numerically this time and we write it in terms of the delta function cross-section as

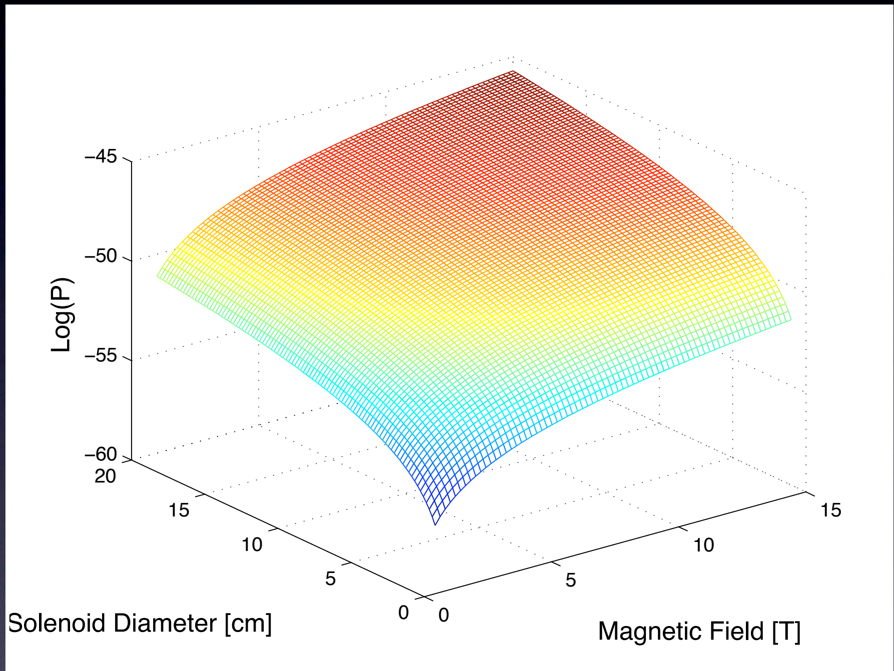
$$\sigma_{tot.}^{well} = \frac{\pi}{32} g^2 B^2 D^4 E \left[\int_0^{2\pi} \left| \frac{J_1(qa)}{qa} \right|^2 d\theta \right] = \sigma_{tot.}^{\delta} \frac{2}{\pi} \left[\int_0^{2\pi} \left| \frac{J_1(qa)}{qa} \right|^2 d\theta \right] = \sigma_{tot.}^{\delta} \frac{2}{\pi} I$$

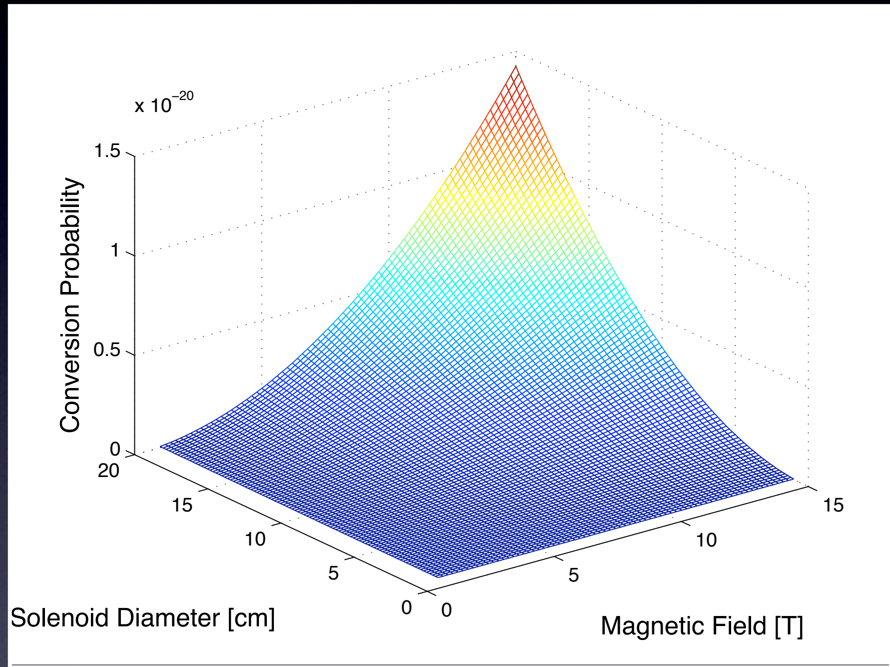
- From the relation $P = \sigma_{tot}/D$ we get also the conversion probability

$$P_{well} = P_\delta \frac{2}{\pi} I = \frac{\pi}{32} g^2 B^2 D^3 E I$$

3 Finite Sized Solenoids

Ideal Solenoid

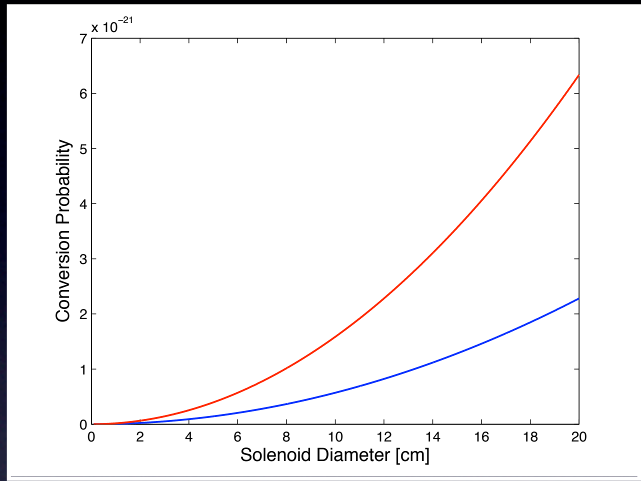




$$P_{well} \propto g^2 B^2 D^2$$

3 Finite Sized Solenoids

Ideal Solenoid



B [Tesla]	D [cm]	$\sigma_{tot.}^{well}$ [cm]	N_{well} [1/sec]	P_{well}
10	1	1.58E-23	5.80E-11	1.58E-23
10	10	1.58E-20	5.80E-08	1.58E-21
6	2	4.56E-23	1.67E-10	2.29E-23
6	20	4.56E-20	1.67E-07	2.29E-21

4 Resonant Scattering

Considering Massive Photons

- The U(1) symmetry holds up whenever the axion mass equals an effective photon mass inside a medium.
- When considering the massive case, the Green's function has to be rewritten with the replacement $1/\sqrt{E} \to 1/((E^2 - m_a^2)^{1/2})^{1/2}$.
- The delta function cross-section will be modified as a consequence to give a new energy dependence

$$\sigma_{tot}^{\delta} \to \frac{\pi g^2 B^2 R^4 E^2}{4\sqrt{(E^2 - m_a^2)}}$$

- A resonance appears whenever $E \sim m_a$.
- Taking the limit of zero momentum means to consider only zero modes in the Fourier transform. Hence, the cross-section of a finite potential becomes of the form of the modified delta function potential.

5 Summary

Comparison with 1D Case

- The conversion probability in 1D is $P_{1D} = g^2 B^2 l^2 / 4$.
- Comparison with the 2D calculations shows increased axion-photon conversion probabilities:

B [Tesla]	D [cm]	P_{1D}	P_{δ}	P_{well}
10	1	5.94E-24	3.08E-15	1.58E-23
10	10	5.94E-22	3.08E-12	1.58E-21
6	2	8.56E-24	8.87E-15	2.29E-23
6	20	8.56E-22	8.87E-12	2.29E-21

5 Summary

Infinitely Thin Solenoid

- The δ function approximation becomes physically realized in the resonant scattering case.
- At the zero momentum case $(k = 0)$, the wavelength is infinite and the ratio between the potential width and the wavelength goes to zero.
- zero momentum implies zero momentum transfer from the relation

$$q^2 = 4k^2 \sin^2(\theta/2)$$

- Therefore, the scattering amplitudes of a finite sized potential become isotropic - similarly to the delta function case.

5 Summary

Future Prospects

- Considering resonant scattering using tunable lasers. Achieving a resonance (i.e. an axion beam with energy equal to the axion mass) depends on how fine the tuning can be made in the lab.
- Considering a quadropole magnetic field, which is more complicated than a the cylindrical symmetric case we have considered but is more accessible as a possible experimental setup.
- Solar magnetic fields (magnetic flux tubes).

Thank You!