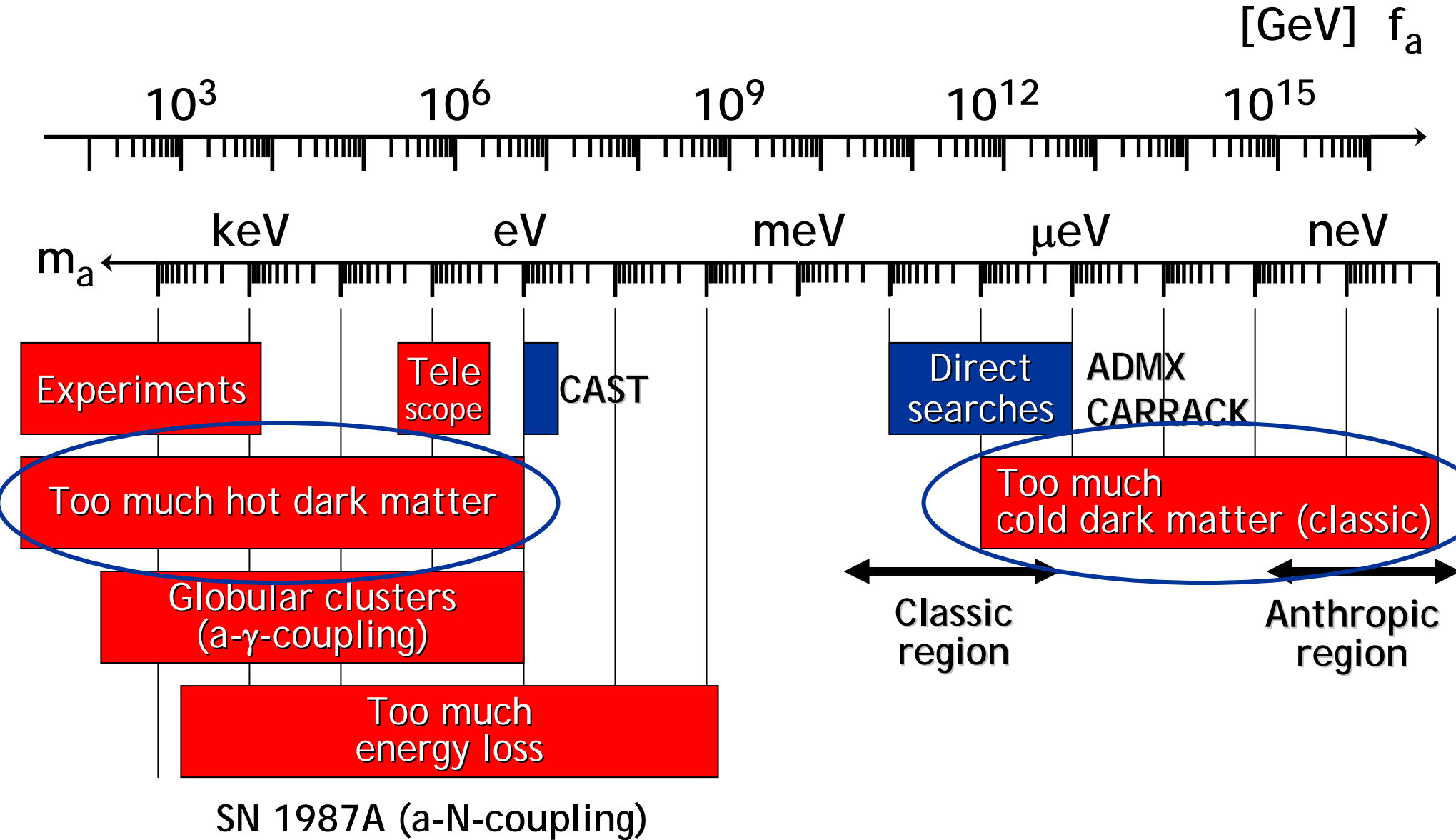


# ASTROPHYSICAL AND COSMOLOGICAL BOUNDS ON AXIONS



STEEN HANNESTAD  
DURHAM, 17 JULY 2009

# Axion Bounds



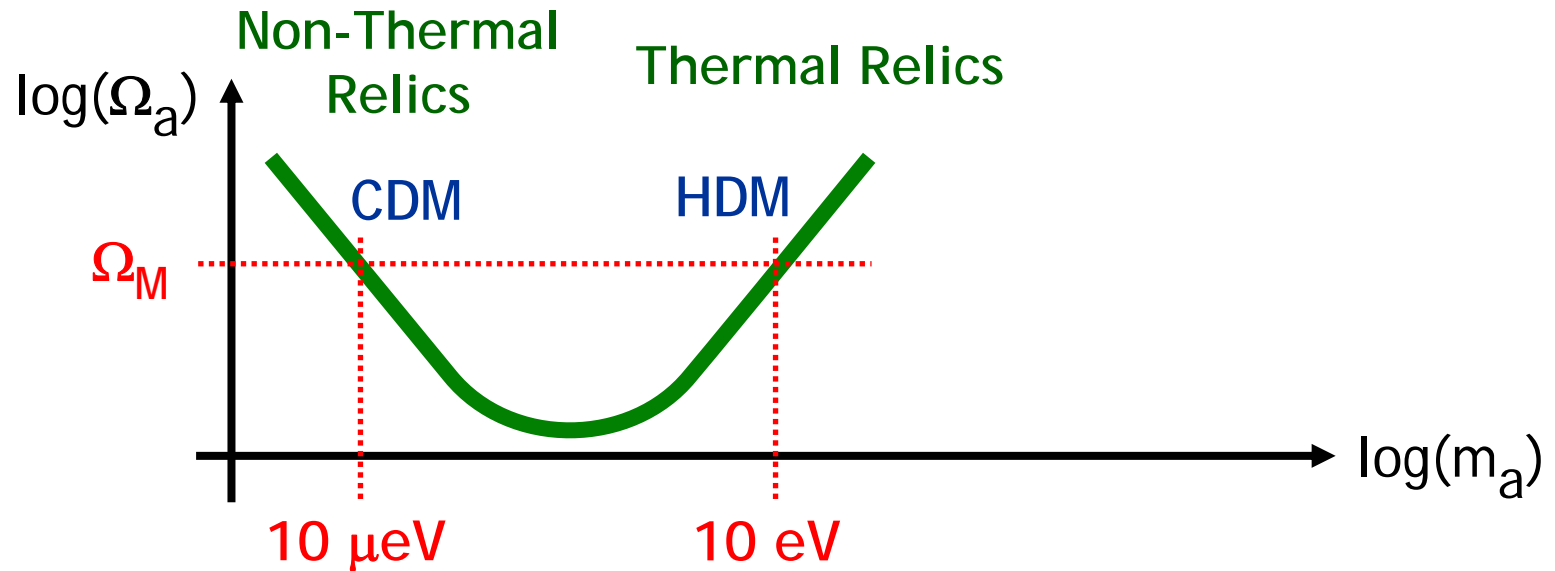
# How are axions created cosmologically

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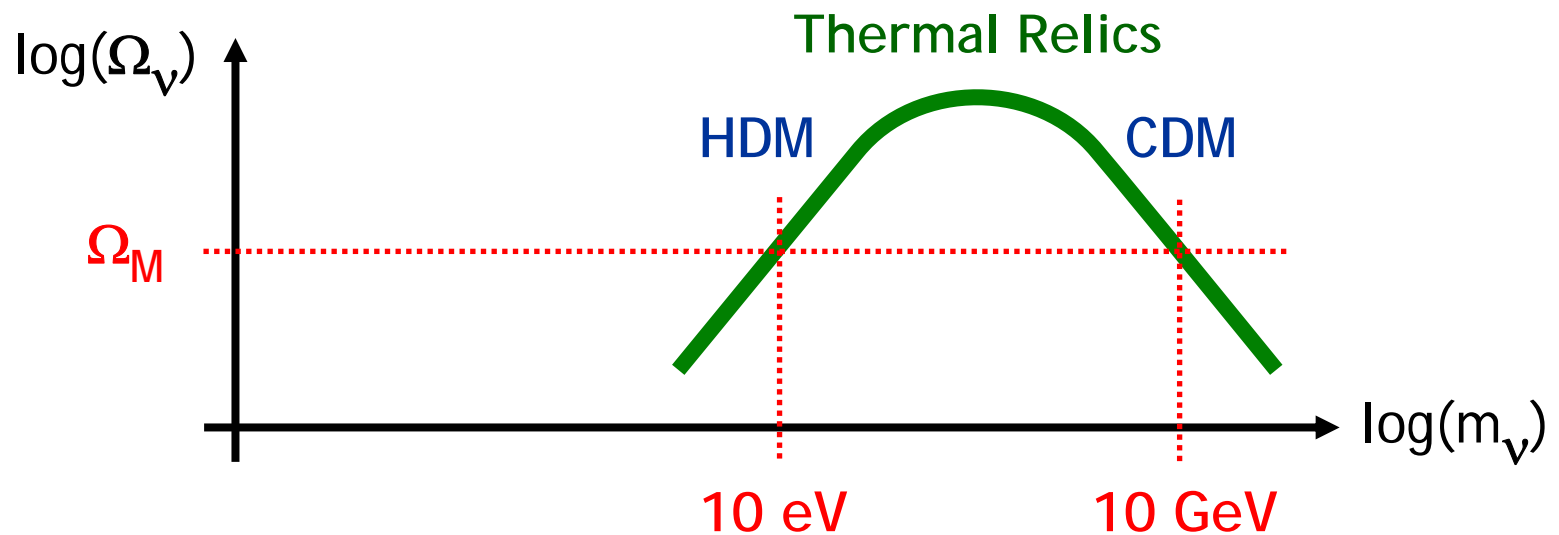
- High mass axions are thermalised at temperatures of order  $\sim \text{MeV} - \text{GeV}$  and act as hot dark matter
- Low mass axions are created non-thermally at high temperature and act as cold dark matter

# Lee-Weinberg Curve for Neutrinos and Axions

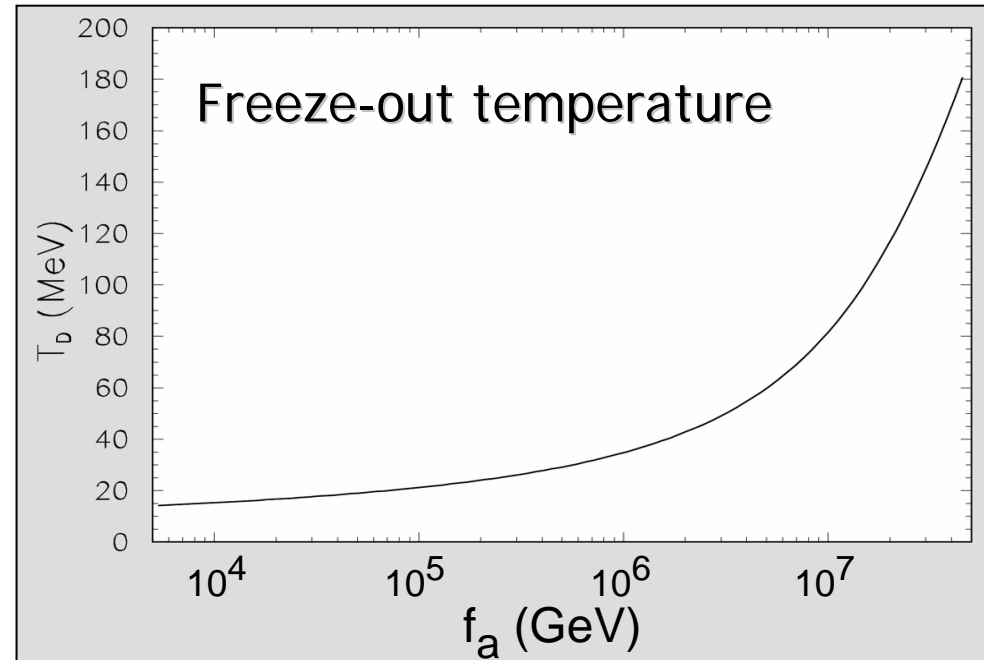
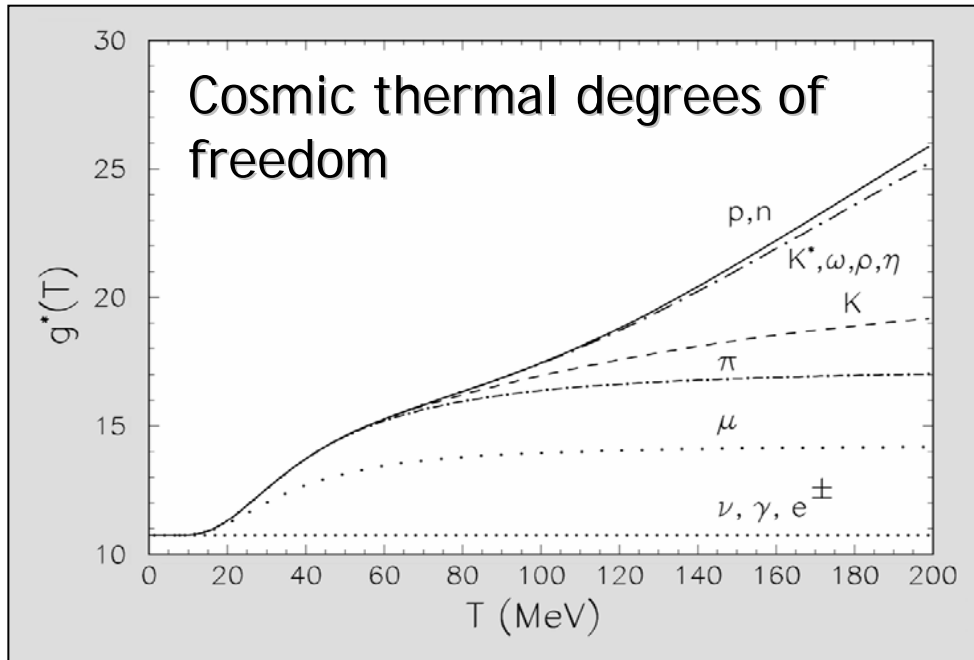
## Axions



## Neutrinos & WIMPs



# Axion Hot Dark Matter from Thermalization after $\Lambda_{\text{QCD}}$

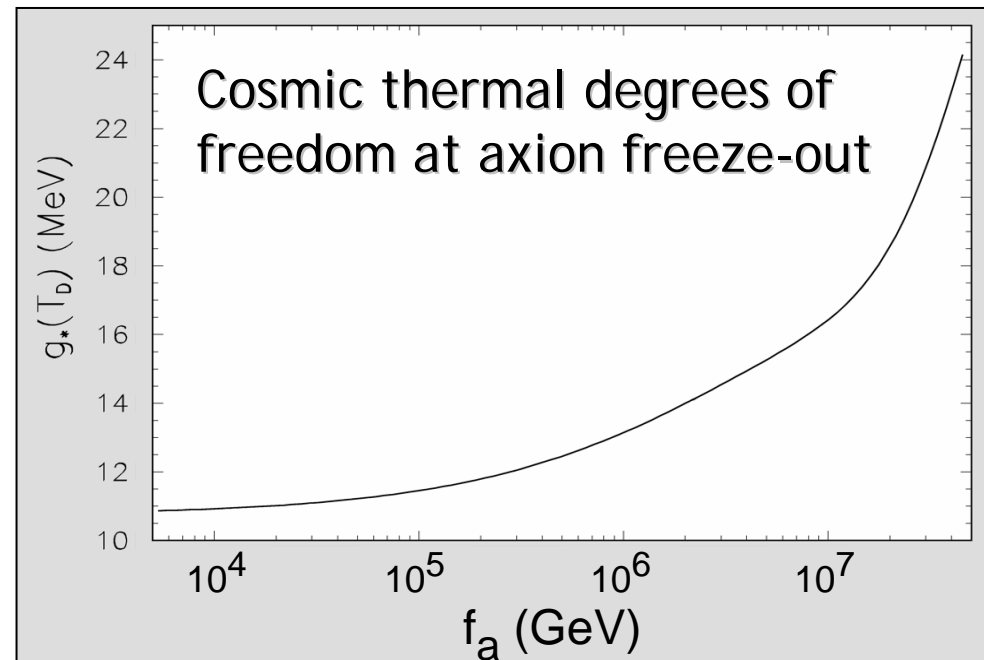


$$\mathcal{L}_{a\pi} = \frac{C_{a\pi}}{f_a f_\pi} (\pi^0 \pi^+ \partial_\mu \pi^- + \pi^0 \pi^- \partial_\mu \pi^+ - 2\pi^+ \pi^- \partial_\mu \pi^0) \partial^\mu a$$

$\begin{array}{cc} \pi & \pi \\ & \times \\ \pi & a \end{array}$

$C_{a\pi} = \frac{1-z}{3(1+z)} \approx 0.094$

Chang & Choi, PLB 316 (1993) 51  
 STH, Mirizzi & Raffelt, JCAP 07 (2005) 02



# Axion HDM properties

AXIONS AFFECT STRUCTURE FORMATION  
BECAUSE THEY ARE A SOURCE OF DARK MATTER

HOWEVER, eV AXIONS ARE DIFFERENT FROM CDM  
BECAUSE THEY FREE STREAM

SCALES SMALLER THAN  $d_{\text{FS}}$  DAMPED AWAY, LEADS TO  
SUPPRESSION OF POWER ON SMALL SCALES

CONTRIBUTION TO DENSITY  $\Omega_X h^2 = \frac{m_X g_X}{183 \text{ eV}} \frac{10.75}{g_{*X}} \times \begin{cases} 1 & \text{for fermions} \\ 4/3 & \text{for bosons} \end{cases}$

FREE-STREAMING LENGTH  $\lambda_{\text{FS}} \sim \frac{20 \text{ Mpc}}{\Omega_X h^2} \left( \frac{T_X}{T_\nu} \right)^4 \left[ 1 + \log \left( 3.9 \frac{\Omega_X}{\Omega_m} \frac{T_\nu^2}{T_X^2} \right) \right]$

# Structure Formation with Hot Dark Matter

Standard  $\Lambda$ CDM Model

Neutrinos with  $\Sigma m_\nu = 6.9$  eV

$Z=32.33$



Structure formation simulated with Gadget code  
Cube size 256 Mpc at zero redshift

Troels Haugbølle, <http://whome.phys.au.dk/~haugboel>

# Available cosmological data

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The cosmic microwave background

Large scale structure of galaxies

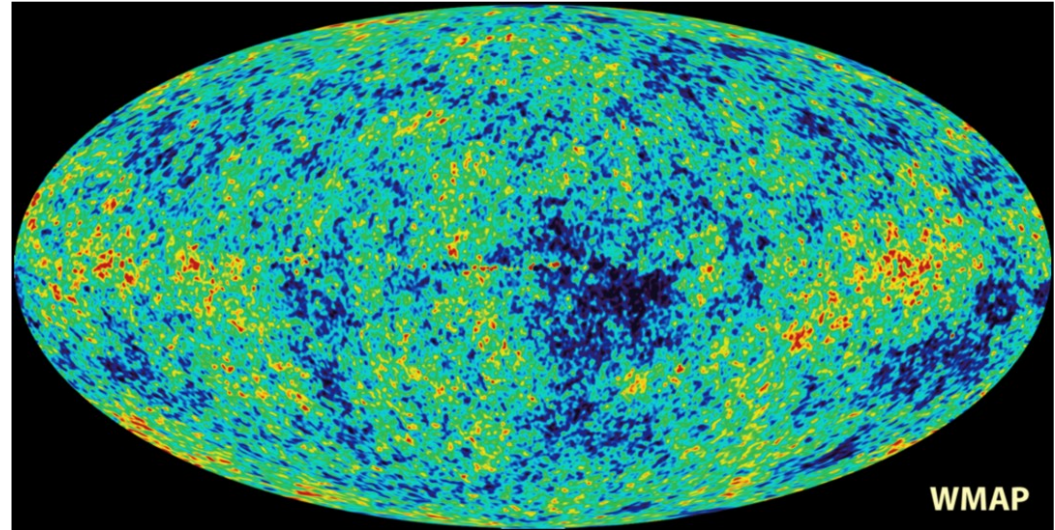
The Lyman-alpha forest

....

# Power Spectrum of CMB Temperature Fluctuations

Sky map of CMBR temperature fluctuations

$$\Delta(\theta, \varphi) = \frac{T(\theta, \varphi) - \langle T \rangle}{\langle T \rangle}$$

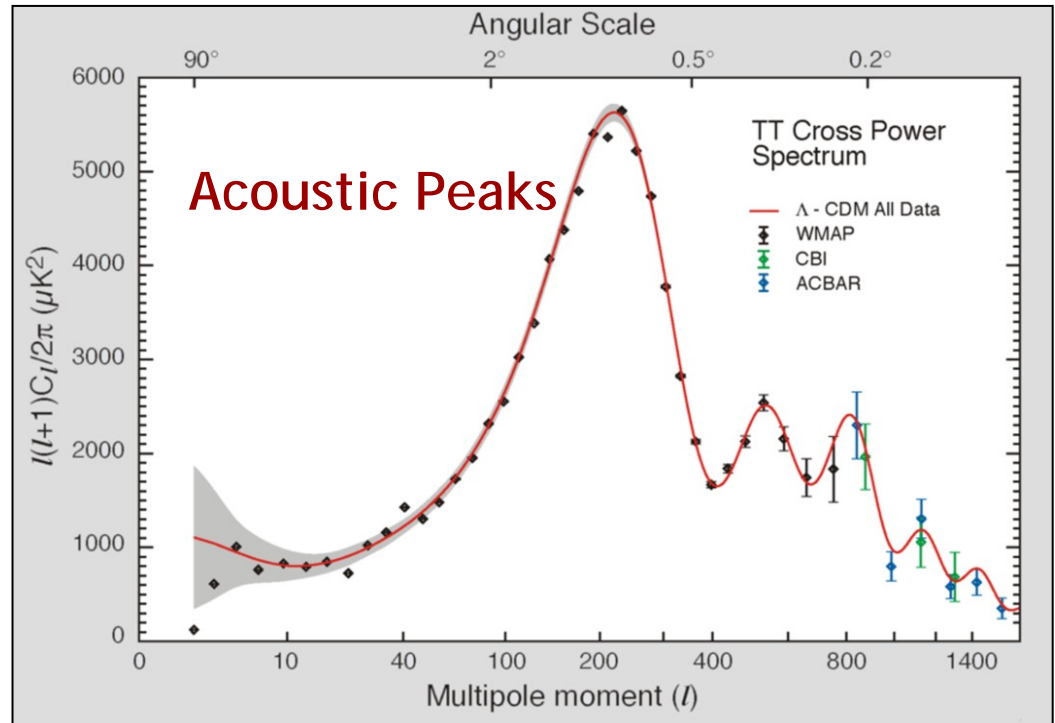


Multipole expansion

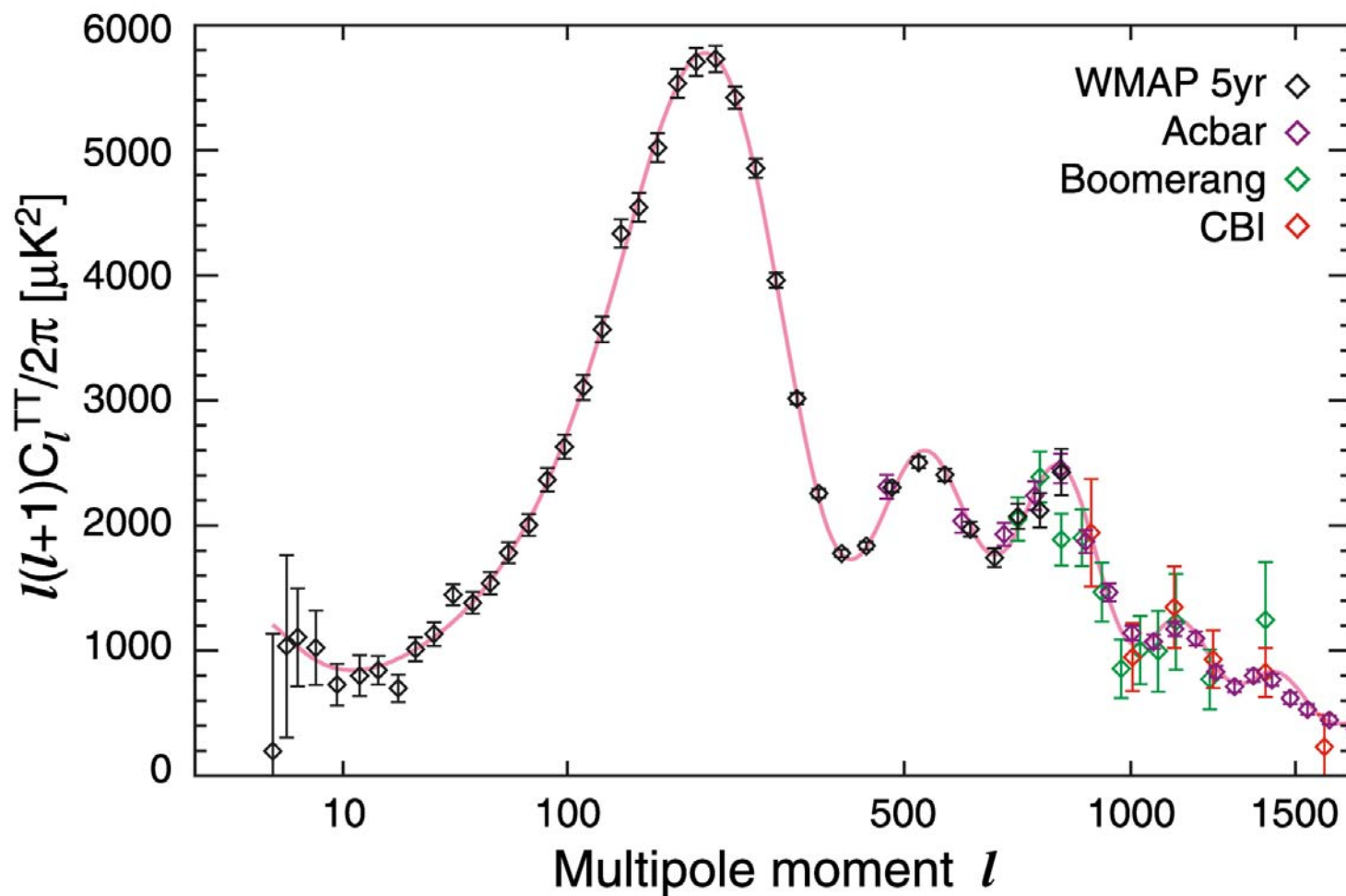
$$\Delta(\theta, \varphi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\theta, \varphi)$$

Angular power spectrum

$$C_{\ell} = \langle a_{\ell m}^* a_{\ell m} \rangle = \frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} a_{\ell m}^* a_{\ell m}$$



# WMAP-5 TEMPERATURE POWER SPECTRUM



M NOLTA ET AL., arXiv:0803.0593

# Power Spectrum of Density Fluctuations

Field of density fluctuations

$$\delta(x) = \frac{\delta\rho(x)}{\bar{\rho}}$$

Fourier transform

$$\delta_k = \int d^3x e^{-ik \cdot x} \delta(x)$$

Power spectrum essentially square of Fourier transformation

$$\langle \delta_k \delta_{k'} \rangle = (2\pi)^3 \hat{\delta}(k - k') P(k)$$

with  $\hat{\delta}$  the  $\delta$ -function

Power spectrum is Fourier transform of two-point correlation function ( $x = x_2 - x_1$ )

$$\begin{aligned} \xi(x) &= \langle \delta(x_2) \delta(x_1) \rangle = \int \frac{d^3k}{(2\pi)^3} e^{ik \cdot x} P(k) \\ &= \int \frac{d\Omega}{4\pi} \frac{dk}{k} e^{ik \cdot x} \underbrace{\frac{k^3 P(k)}{2\pi^2}}_{\Delta^2(k)} \end{aligned}$$

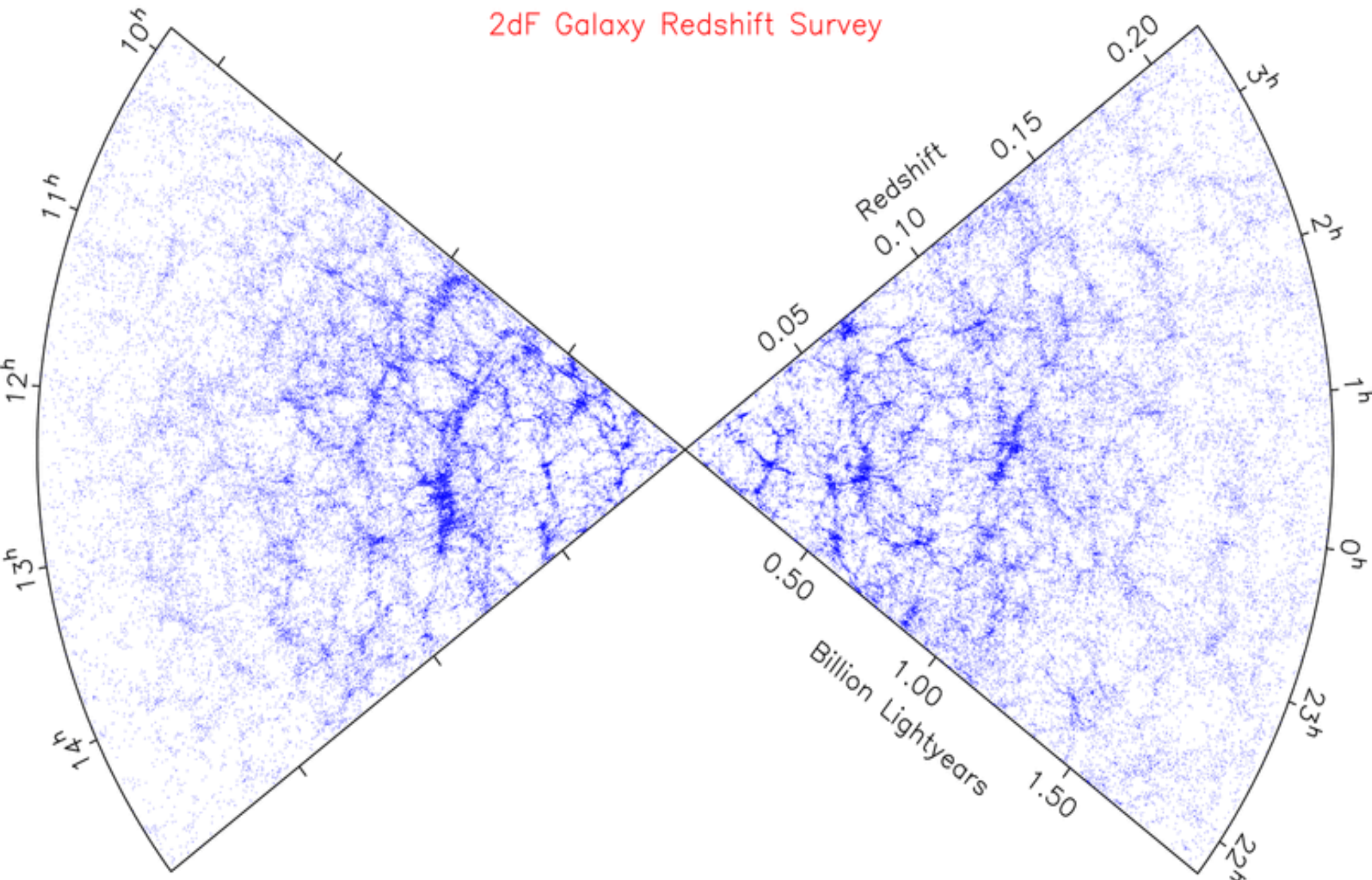
Gaussian random field (phases of Fourier modes  $\delta_k$  uncorrelated) is fully characterized by the power spectrum

$$P(k) = |\delta_k|^2$$

or equivalently by

$$\Delta(k) = \left( \frac{k^3 P(k)}{2\pi^2} \right)^{1/2} = \frac{k^{3/2} |\delta_k|}{\sqrt{2\pi}}$$

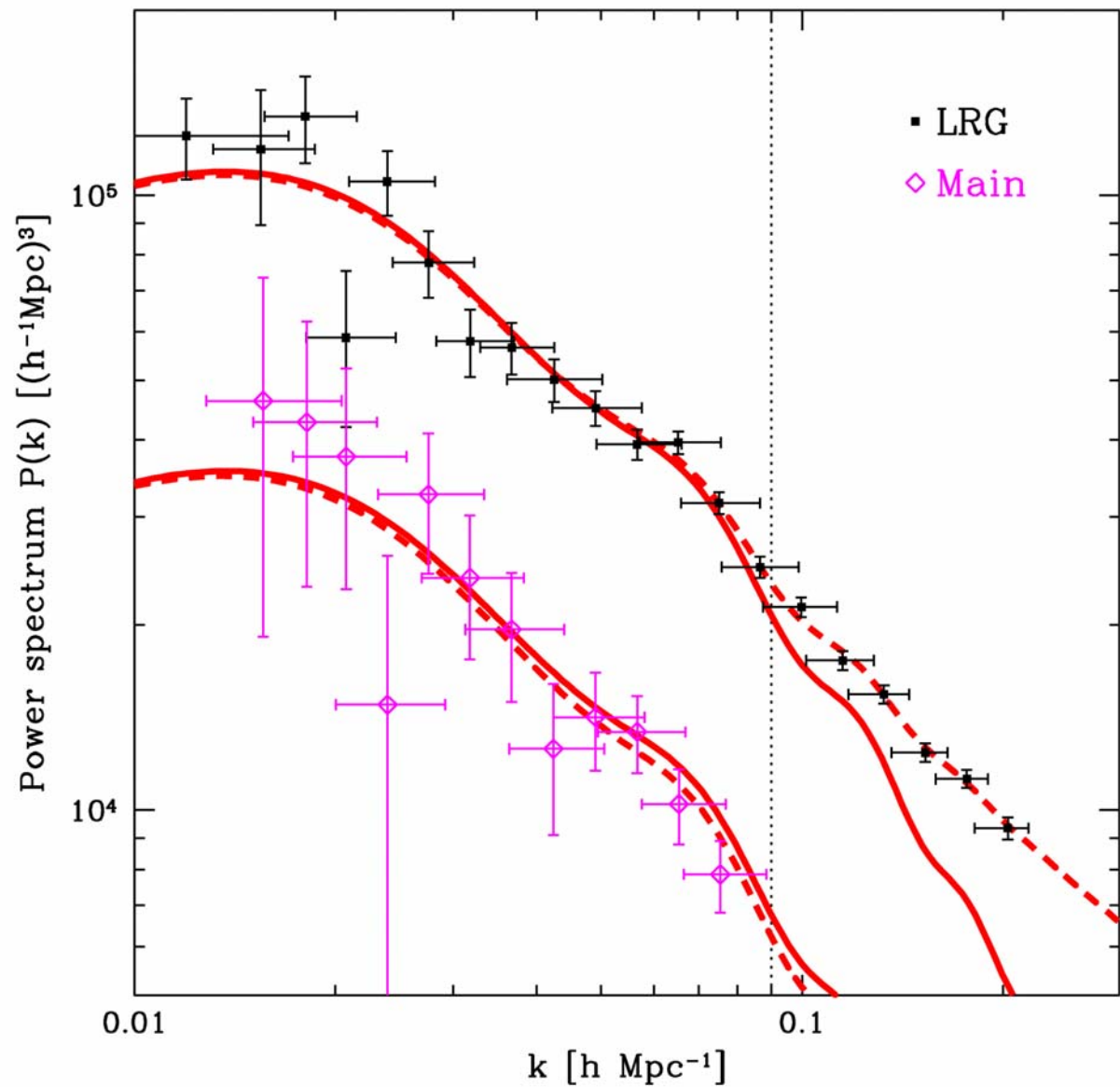
# LARGE SCALE STRUCTURE SURVEYS - 2dF AND SDSS



# SDSS SPECTRUM

## TEGMARK ET AL. 2006

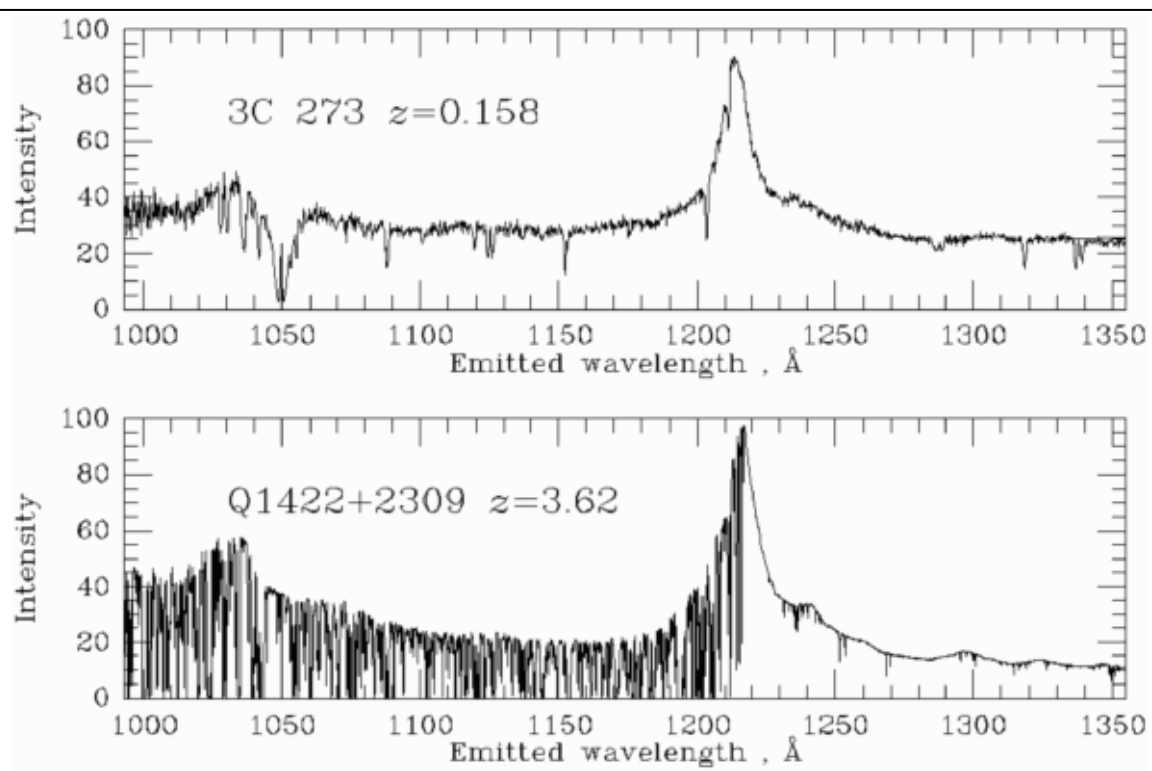
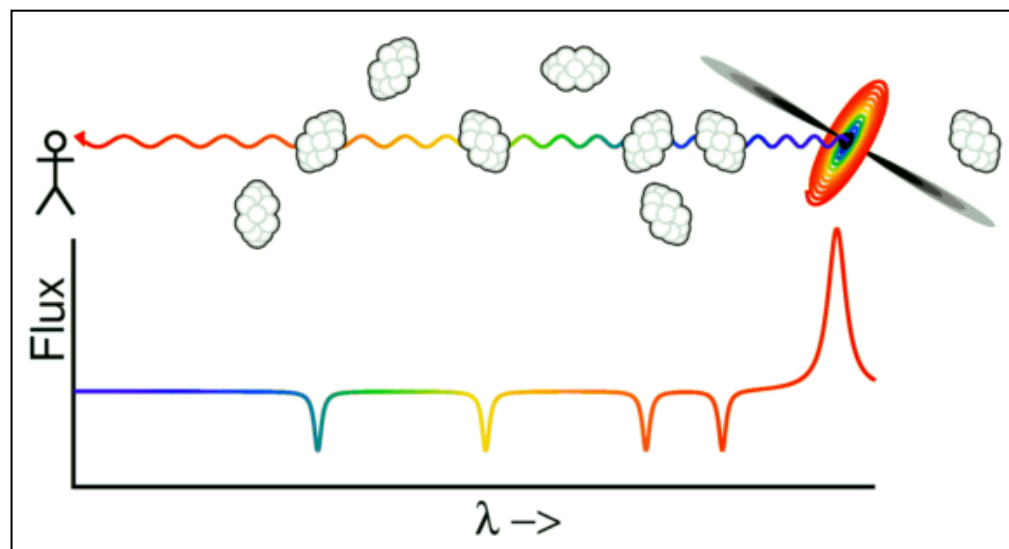
astro-ph/0608632



# Lyman-alpha Forest

- Hydrogen clouds absorb from QSO continuum emission spectrum
- Absorption dips at Ly- $\alpha$  wavelength corresponding to redshift

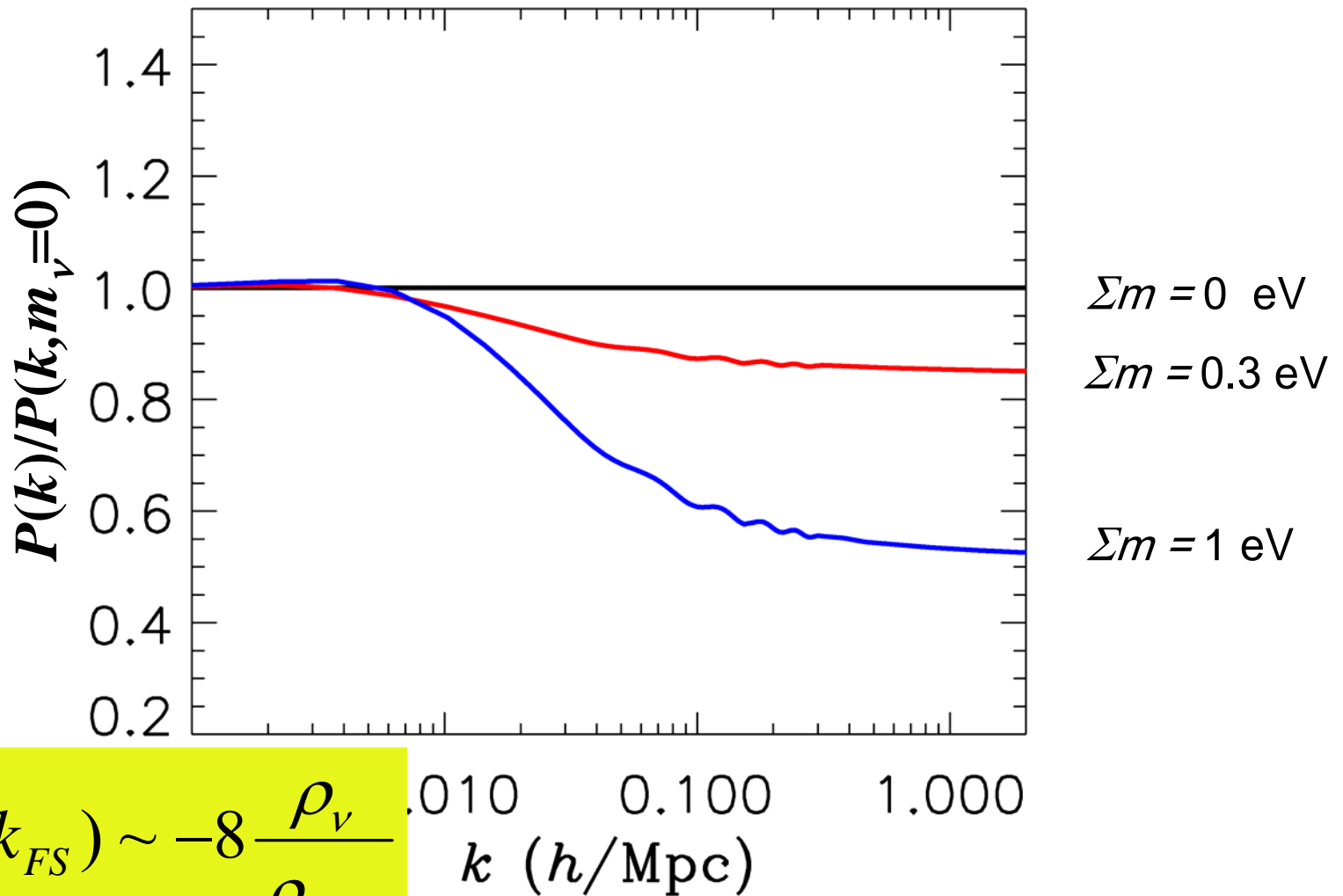
[www.astro.ucla.edu/~wright/Lyman-alpha-forest.html](http://www.astro.ucla.edu/~wright/Lyman-alpha-forest.html)



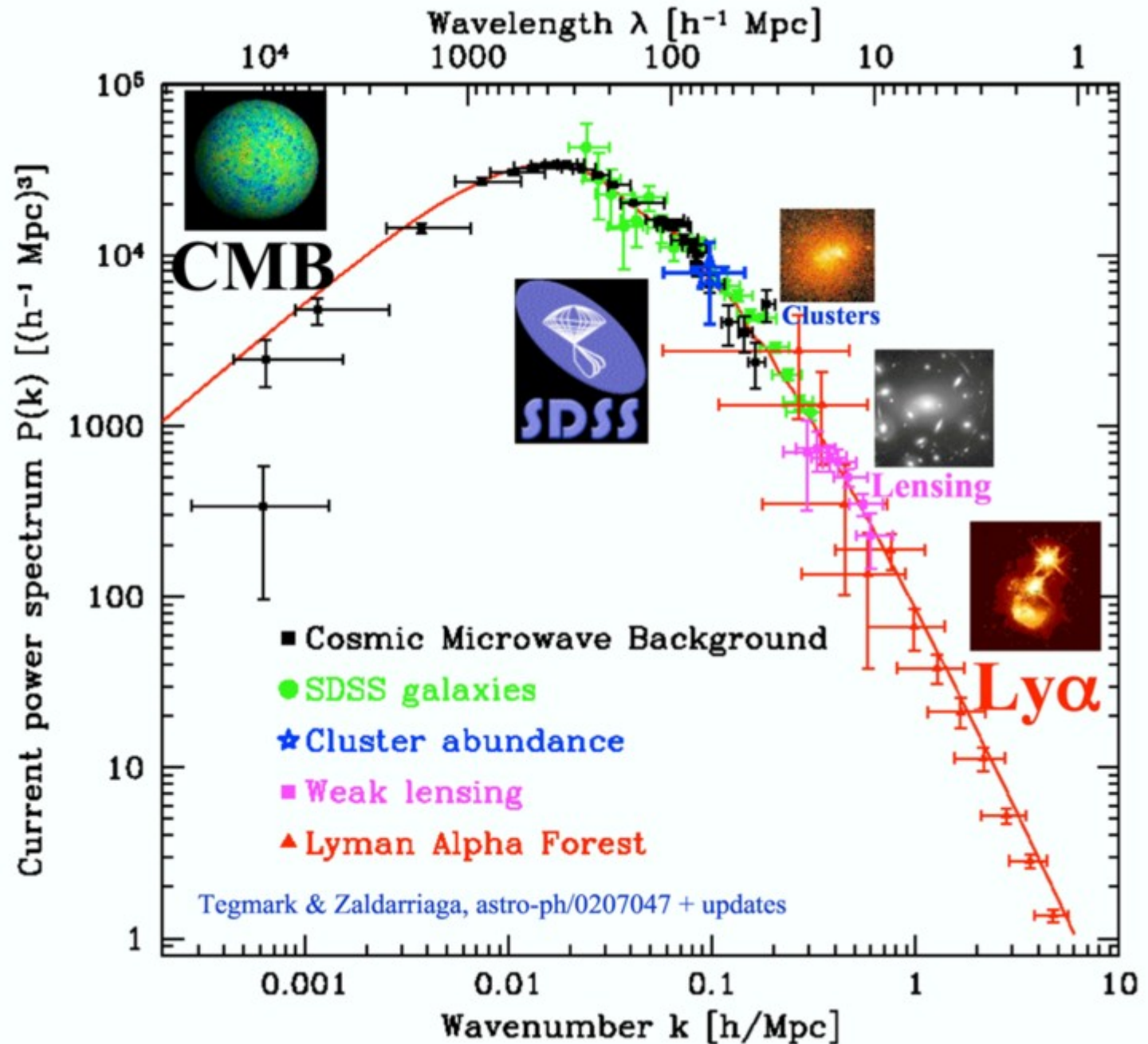
Examples for Lyman- $\alpha$  forest in low- and high-redshift quasars

<http://www.astr.ua.edu/keel/agn/forest.gif>

FINITE NEUTRINO MASSES SUPPRESS THE MATTER POWER SPECTRUM ON SCALES SMALLER THAN THE FREE-STREAMING LENGTH, THE SAME IS TRUE FOR AXIONS



# Power Spectrum of Cosmic Density Fluctuations

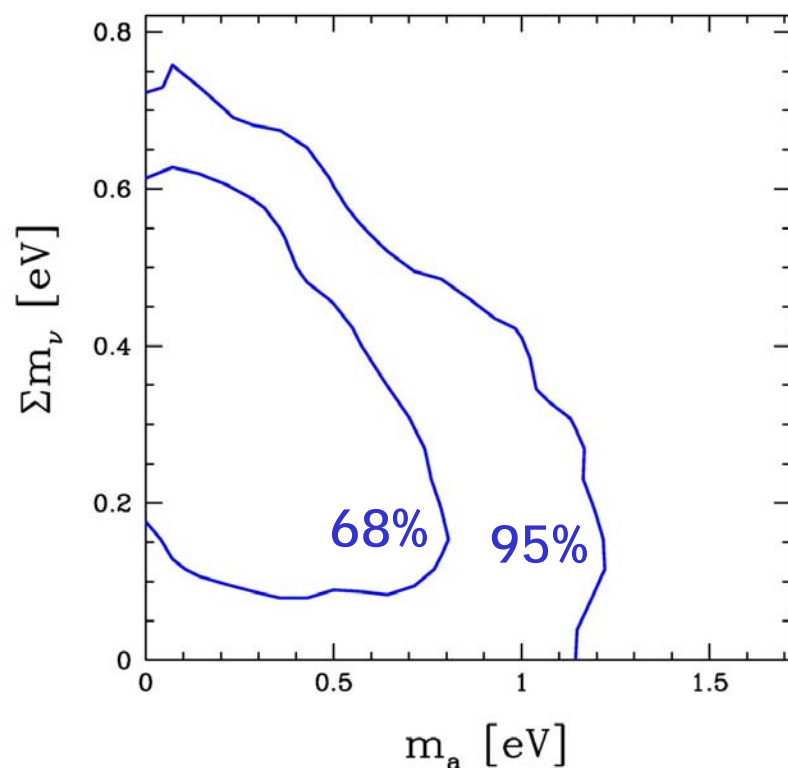


Max Tegmark  
Univ. of Pennsylvania  
max@physics.upenn.edu  
TAUP 2003  
September 5, 2003

# Some Recent Cosmological Limits on Neutrino Masses

	$\Sigma m_\nu / \text{eV}$ (limit 95%CL)	Data / Priors
Hannestad 2003 [astro-ph/0303076]	1.01	WMAP-1, CMB, 2dF, HST
Spergel et al. (WMAP) 2003 [astro-ph/0302209]	0.69	WMAP-1, 2dF, HST, $\sigma_8$
Crotty et al. 2004 [hep-ph/0402049]	1.0 0.6	WMAP-1, CMB, 2dF, SDSS & HST, SN
Hannestad 2004 [hep-ph/0409108]	0.65	WMAP-1, SDSS, SN Ia gold sample, Ly- $\alpha$ data from Keck sample
Seljak et al. 2004 [astro-ph/0407372]	0.42	WMAP-1, SDSS, Bias, Ly- $\alpha$ data from SDSS sample
Hannestad et al. 2006 [astro-ph/0602155]	0.30	WMAP-1, CMB-small, SDSS, 2dF, SN Ia, BAO (SDSS), Ly- $\alpha$ (SDSS)
Spergel et al. 2006 [astro-ph/0603499]	0.68	WMAP-3, SDSS, 2dF, SN Ia, $\sigma_8$
Seljak et al. 2006 [astro-ph/0604335]	0.14	WMAP-3, CMB-small, SDSS, 2dF, SN Ia, BAO (SDSS), Ly- $\alpha$ (SDSS)

# Axion Hot Dark Matter Limits from Precision Data



Credible regions for neutrino plus axion hot dark matter (WMAP-5, LSS, BAO, SNIa)  
Hannestad, Mirizzi, Raffelt & Wong  
[arXiv:0803.1585]

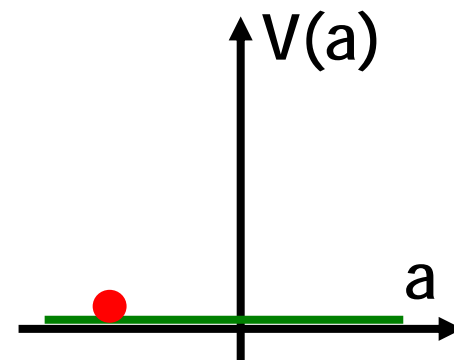
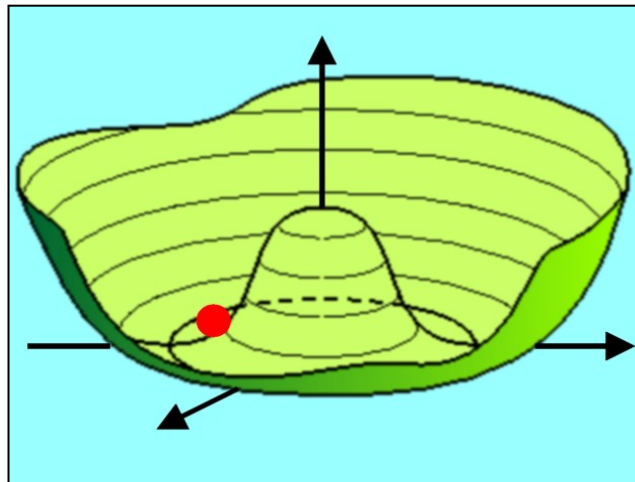
## Marginalizing over unknown neutrino hot dark matter component

$m_a < 1.0 \text{ eV (95\% CL)}$	WMAP-5, LSS, BAO, SNIa	Hannestad, Mirizzi, Raffelt & Wong [arXiv:0803.1585]
$m_a < 0.4 \text{ eV (95\% CL)}$	WMAP-3, small-scale CMB, HST, BBN, LSS, Ly- $\alpha$	Melchiorri, Mena & Slosar [arXiv:0705.2695]

# Creation of Cold Cosmological Axions

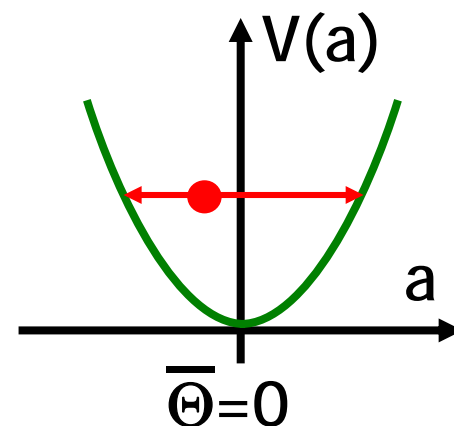
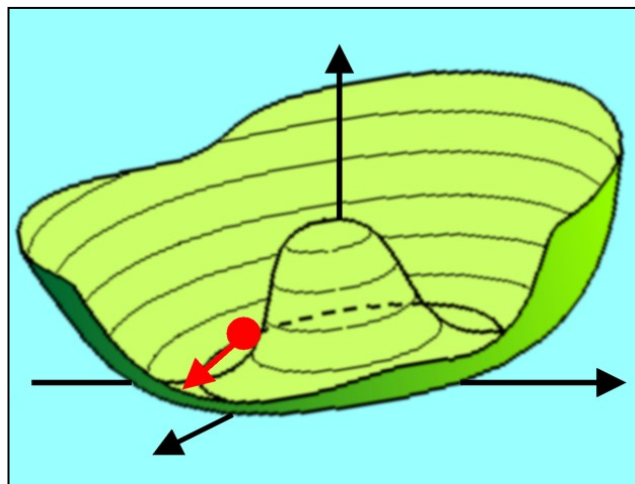
$T \sim f_a$  (very early universe)

- $U_{PQ}(1)$  spontaneously broken
- Higgs field settles in “Mexican hat”
- Axion field sits fixed at  $a_1 = \Theta_1 f_a$



$T \sim 1 \text{ GeV}$  ( $H \sim 10^{-9} \text{ eV}$ )

- Axion mass turns on quickly by thermal instanton gas
- Field starts oscillating when  $m_a \gtrsim 3H$
- Classical field oscillations (axions at rest)
- Axion number density in comoving volume conserved



$$n_a R^3 = m_a(T_1) a_1^2 R_1^3 \sim 3H_1 R_1^3 \Theta_1^2 f_a^2$$

- Axion mass density today:  $\rho_a = m_a n_a \propto \Theta_1^2 m_a f_a^2 \propto \Theta_1^2 \frac{m_a^2 f_a^2}{m_a} \propto \Theta_1^2 \frac{m_\pi^2 f_\pi^2}{m_a}$

# Cosmic Axion Density

Modern values for QCD parameters and temperature-dependent axion mass imply (Bae, Huh & Kim, arXiv:0806.0497)

$$\Omega_a h^2 = 0.195 \Theta_i^2 \left( \frac{f_a}{10^{12} \text{ GeV}} \right)^{1.184} = 0.105 \Theta_i^2 \left( \frac{10 \mu\text{eV}}{m_a} \right)^{1.184}$$

If axions provide the cold dark matter:  $\Omega_a h^2 = 0.11$

$$\Theta_i = 0.75 \left( \frac{10^{12} \text{ GeV}}{f_a} \right)^{0.592} = 1.0 \left( \frac{m_a}{10 \mu\text{eV}} \right)^{0.592}$$

- $\Theta_i \sim 1$  implies  $f_a \sim 10^{12} \text{ GeV}$  and  $m_a \sim 10 \mu\text{eV}$  ("classic window")
- $f_a \sim 10^{16} \text{ GeV}$  (GUT scale) or larger (string inspired) requires  $\Theta_i \lesssim 0.003$  ("anthropic window")

# Cold Axion Populations

## Case 1:

Inflation after PQ symmetry breaking

Homogeneous mode oscillates after

$$T \lesssim \Lambda_{\text{QCD}}$$

Dependence on initial misalignment angle

$$\Omega_a \propto \Theta_i^2$$

Dark matter density a cosmic random number (“environmental parameter”)

- Isocurvature fluctuations from large quantum fluctuations of massless axion field created during inflation
- Strong CMB bounds on isocurvature fluctuations
- Scale of inflation required to be small

## Case 2:

Reheating restores PQ symmetry

- Cosmic strings of broken  $U_{\text{PQ}}(1)$  form by Kibble mechanism
- Radiate long-wavelength axions
- $\Omega_a$  independent of initial conditions

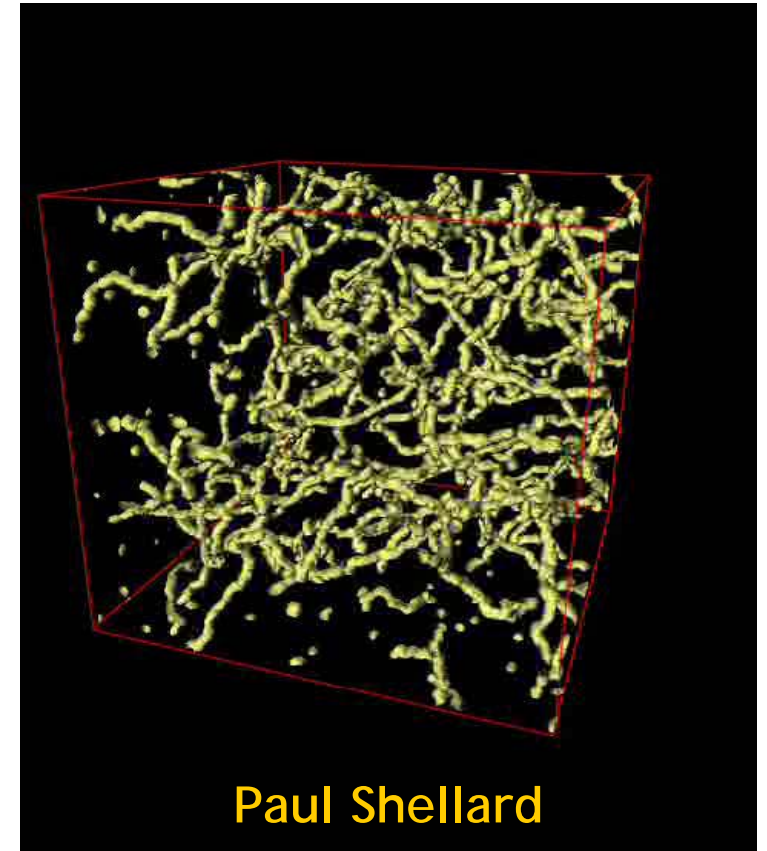
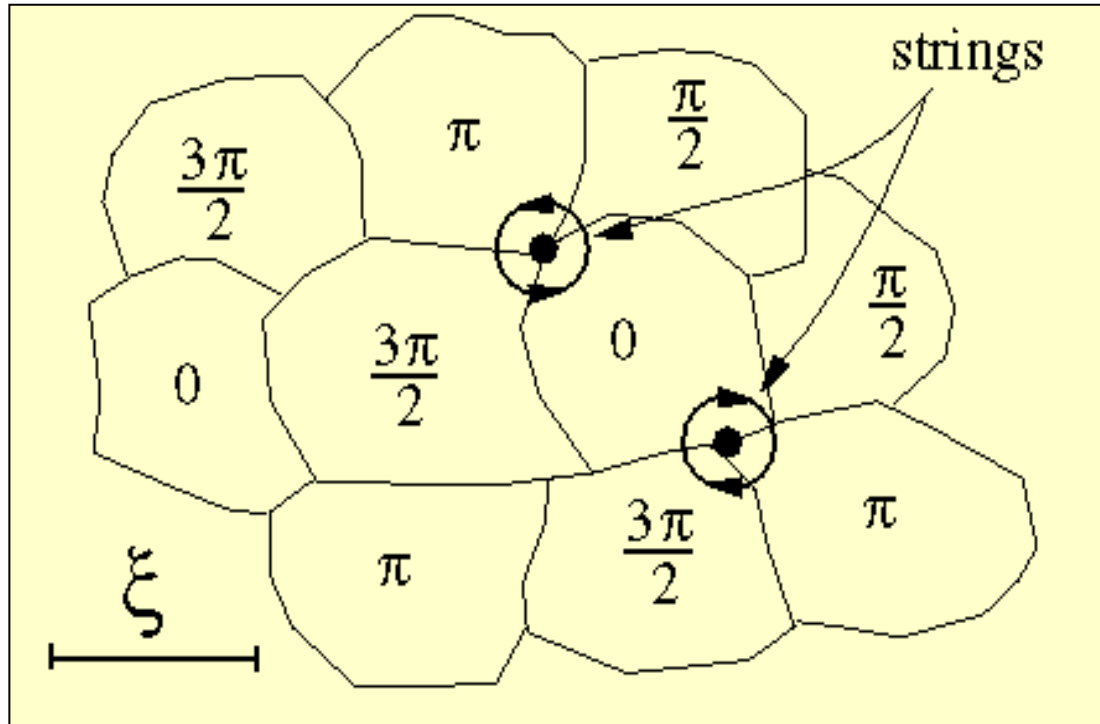
Inhomogeneities of axion field large, self-couplings lead to formation of mini-clusters

Typical properties

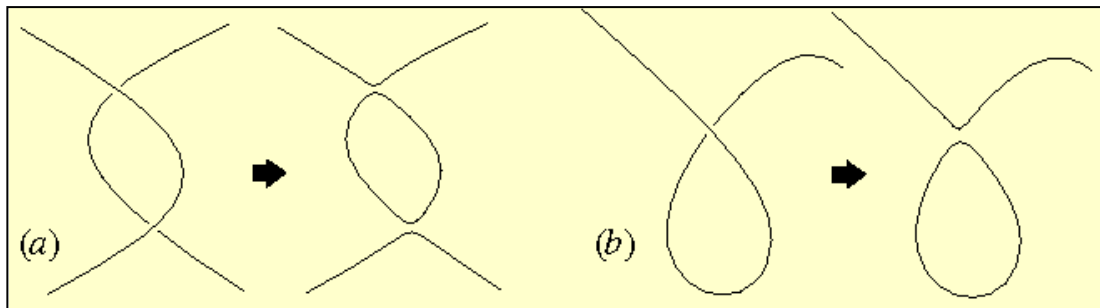
- Mass  $\sim 10^{-12} M_{\text{sun}}$
- Radius  $\sim 10^{10} \text{ cm}$
- Mass fraction up to several 10%

# Axions from Cosmic Strings

Strings form by Kibble mechanism  
after break-down of  $U_{PQ}(1)$



Small loops form by self-intersection



# Inflation, Axions, and Anthropic Selection

---

If PQ symmetry is not restored after inflation

- Axion density determined by initial random number  $-\pi < \Theta_i < +\pi$
- Different in different patches of the universe
- Our visible universe, after inflation, from a single patch
- Axion/photon ratio a cosmic random number, chosen by spontaneous symmetry breaking process

Allows for small  $\Theta_i \lesssim 0.003$  and thus for  $f_a$  at GUT or string scale

- Is this “unlikely” or “unnatural” or “fine tuning”?
- Should one design experiments for very small-mass axion dark matter?

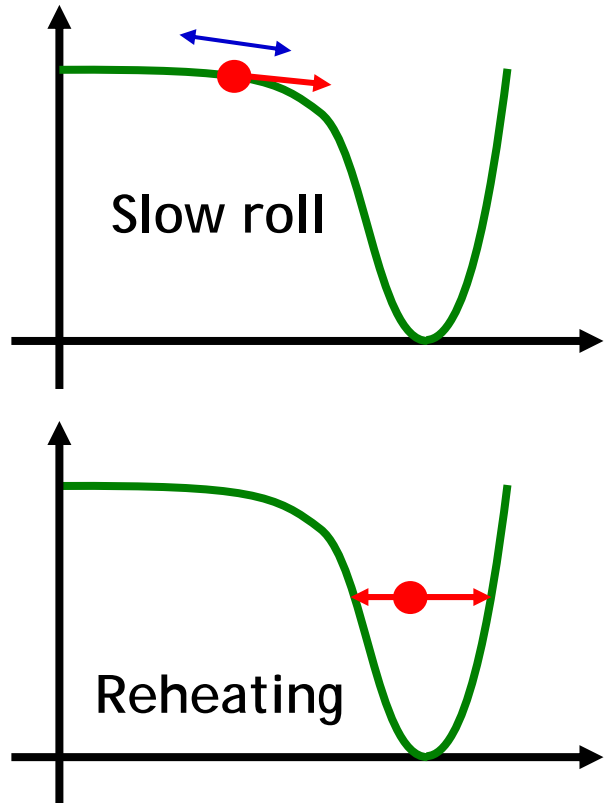
Difficult to form baryonic structures if baryon/dark matter density too low, posterior probability for small  $\Theta_i$  not necessarily small

- Linde, “Inflation and axion cosmology,” PLB 201:437, 1988
- Tegmark, Aguirre, Rees & Wilczek, “Dimensionless constants, cosmology and other dark matters,” PRD 73:023505, 2006 [astro-ph/0511774]

# Creation of Adiabatic vs. Isocurvature Perturbations

## Inflaton field:

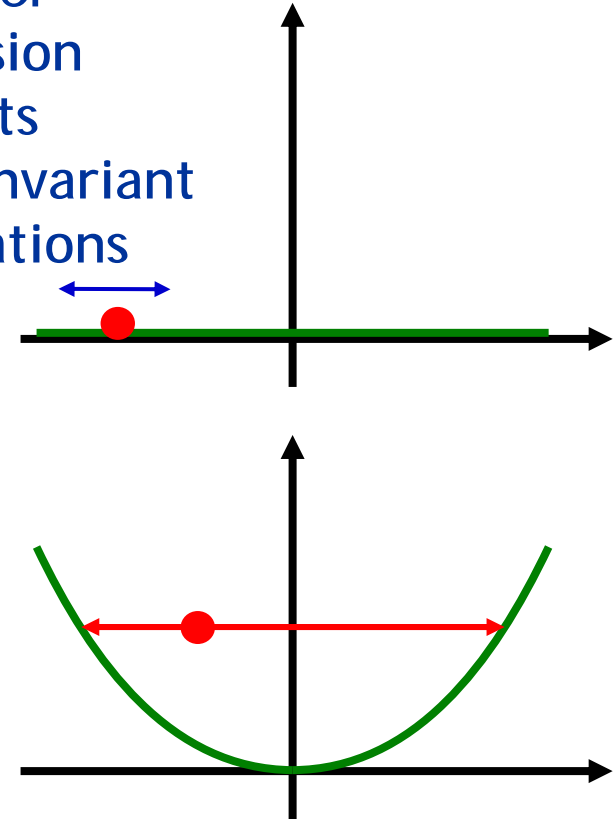
De Sitter expansion imprints  
scale invariant fluctuations



Inflaton decay  $\rightarrow$  matter & radiation  
Fluctuations in both (adiabatic)

## Axion field:

De Sitter  
expansion  
imprints  
scale invariant  
fluctuations



Inflaton decay  $\rightarrow$  radiation  
Axion field oscillates late  $\rightarrow$  matter  
Fluctuations of matter relative to  
radiation: Entropy fluctuations

# Amplitudes of Adiabatic and Isocurvature Perturbations

Entropy fluctuations induced by de Sitter expansion on axion field

$$S(k) = \frac{\Theta^2 - \langle \Theta^2 \rangle}{\langle \Theta^2 \rangle}$$

Isocurvature power spectrum, assuming Gaussian fluctuations ( $n_{\text{iso}} = 1 - 2\varepsilon$ , slow-roll parameter  $\varepsilon$ )

$$\langle |S(k)|^2 \rangle \sim \sigma_\Theta^2 \sim \frac{H_I^2}{\pi^2 f_a^2 \Theta_i^2} \propto \left( \frac{k}{k_0} \right)^{n_{\text{iso}} - 1}$$

Usual curvature power spectrum

$$\langle |R(k)|^2 \rangle \sim \frac{H_I^2}{\pi M_{\text{Pl}}^2 \varepsilon} \propto \left( \frac{k}{k_0} \right)^{n_{\text{ad}} - 1}$$

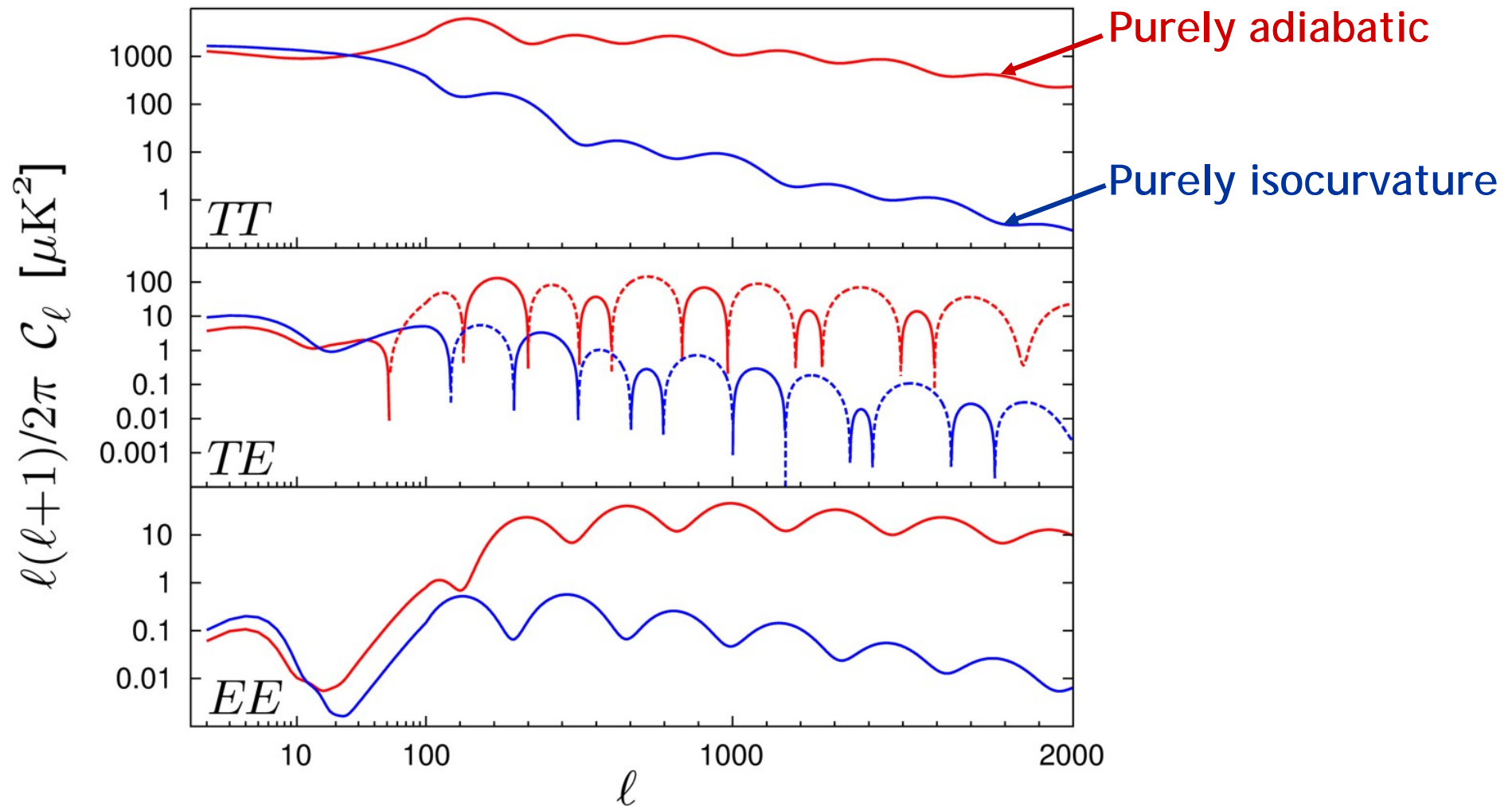
Total power spectrum  
uncorrelated sum

$$P(k) = \langle |R(k)|^2 \rangle + \langle |S(k)|^2 \rangle$$

Isocurvature fraction  
at pivot scale  $k_0 = 0.002 \text{ Mpc}^{-1}$

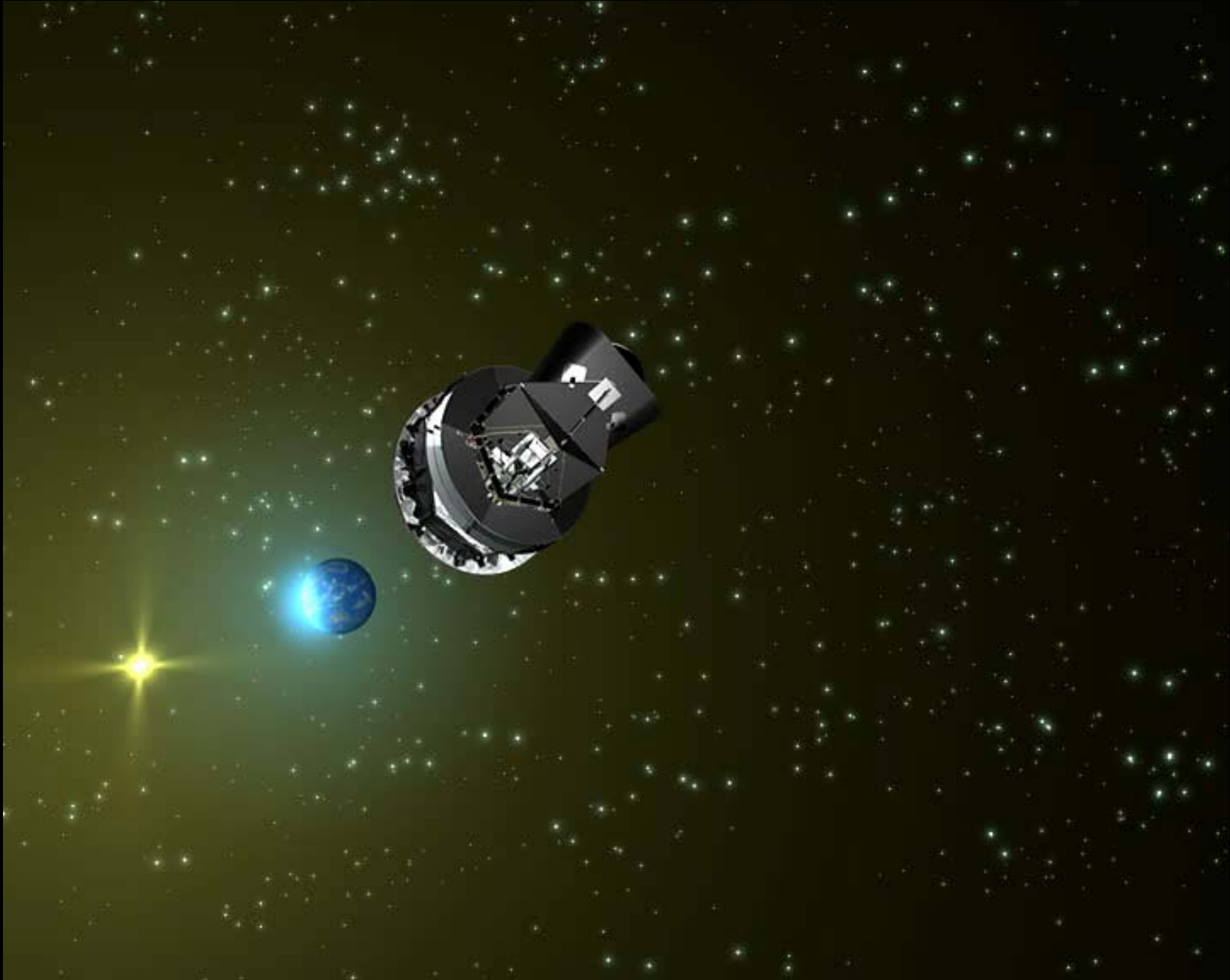
$$\alpha = \frac{\langle |S(k)|^2 \rangle}{\langle |R(k)|^2 \rangle + \langle |S(k)|^2 \rangle} \bigg|_{k=k_0} \sim \frac{H_I^2}{A_S \pi^2 f_a^2 \Theta_i^2}$$

# CMB Angular Power Spectrum

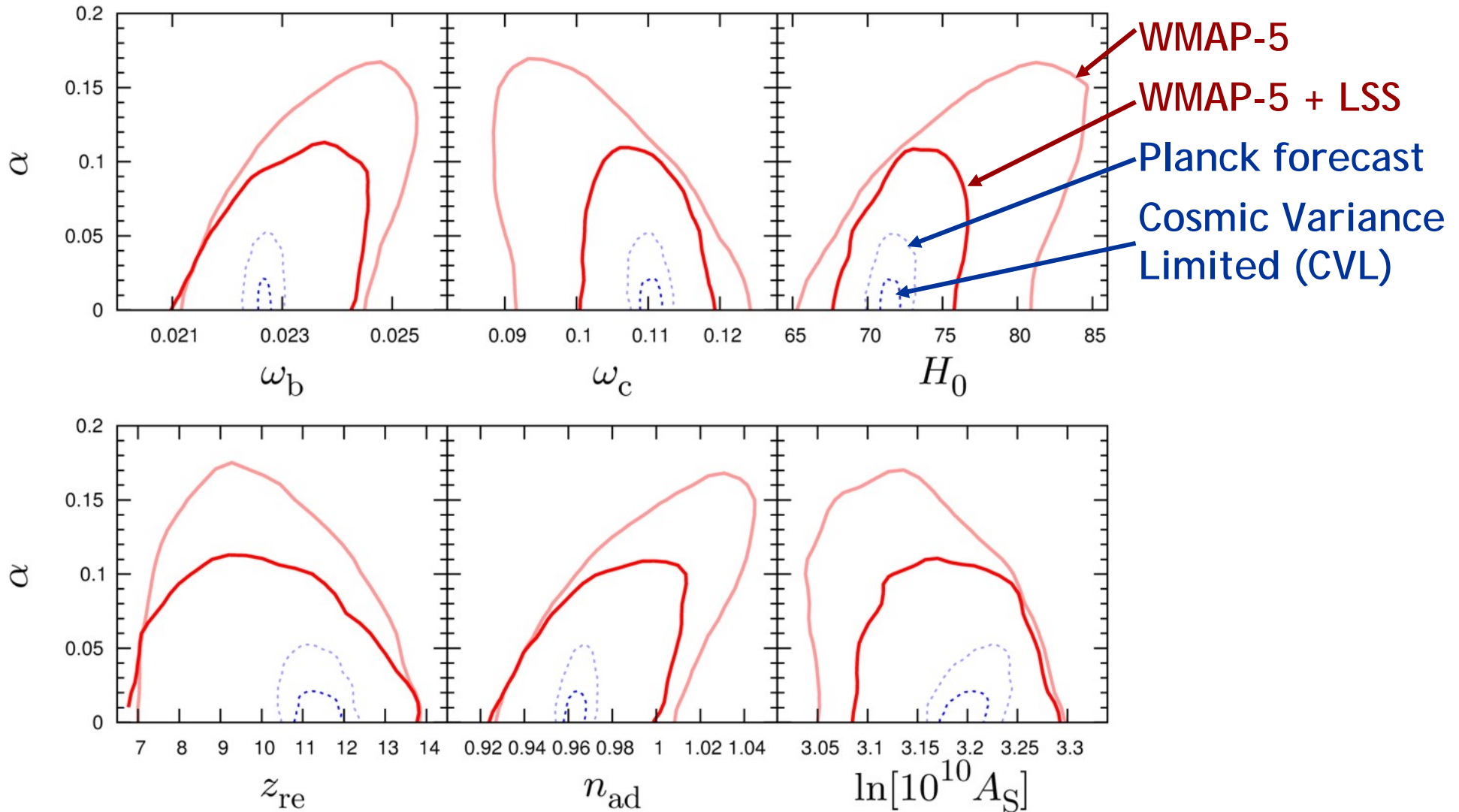


Hamann, Hannestad, Raffelt & Wong, arXiv:0904.0647

# PLANCK Satellite – Successful Launch on 14 May 2009

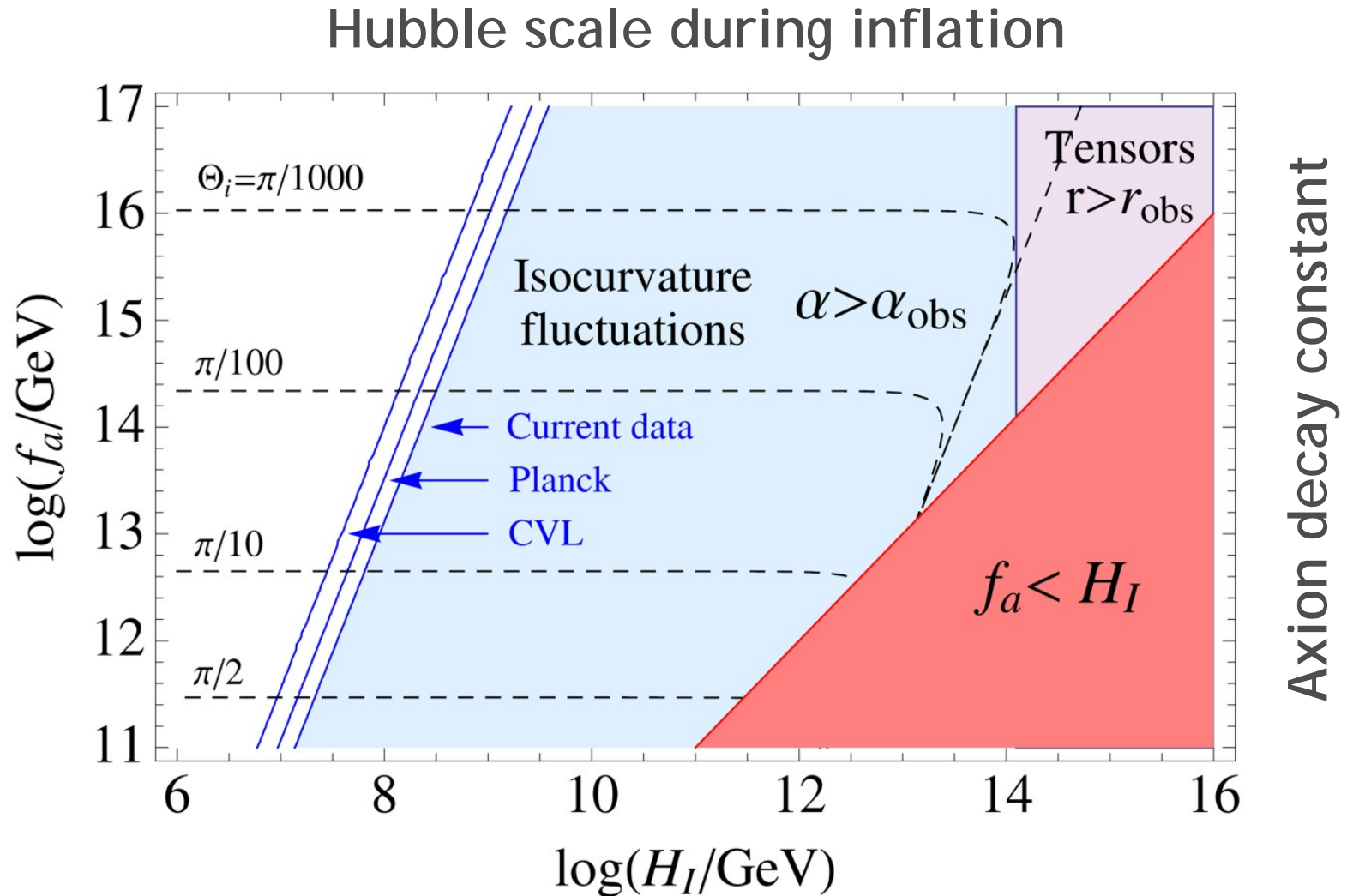


# Parameter Degeneracies



Hamann, Hannestad, Raffelt & Wong, arXiv:0904.0647

# Isocurvature Forecast



Hamann, Hannestad, Raffelt & Wong, arXiv:0904.0647

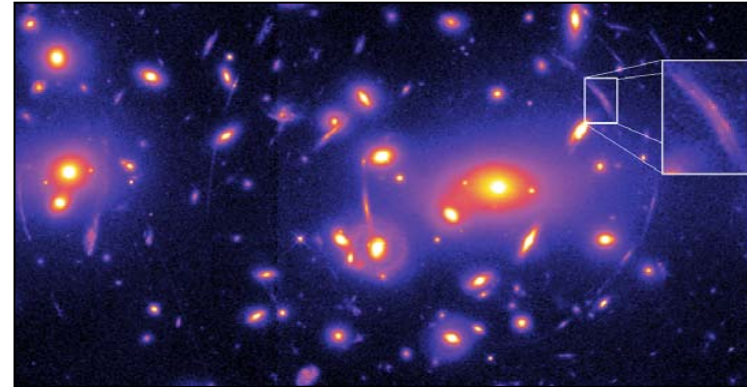
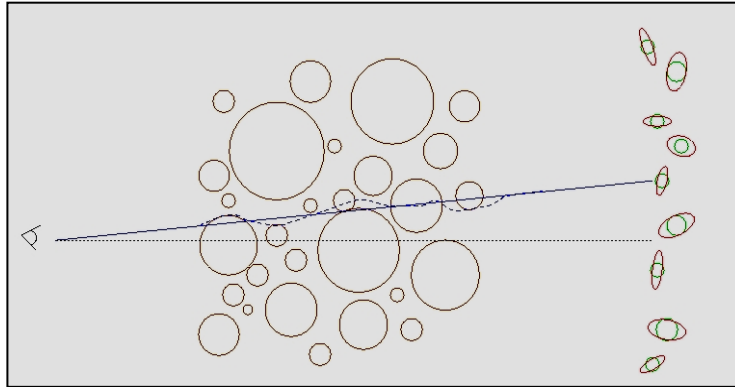
# What will happen wrt axion HDM in the future

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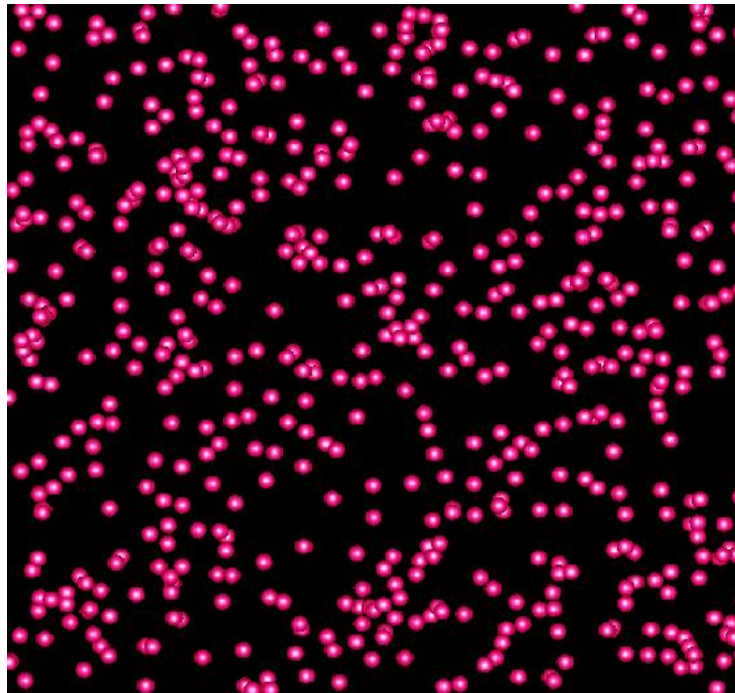
A large amount of new precision data will become available in the next decade

- New CMB data
- Weak lensing
- Baryon acoustic oscillation surveys
- 21-cm surveys

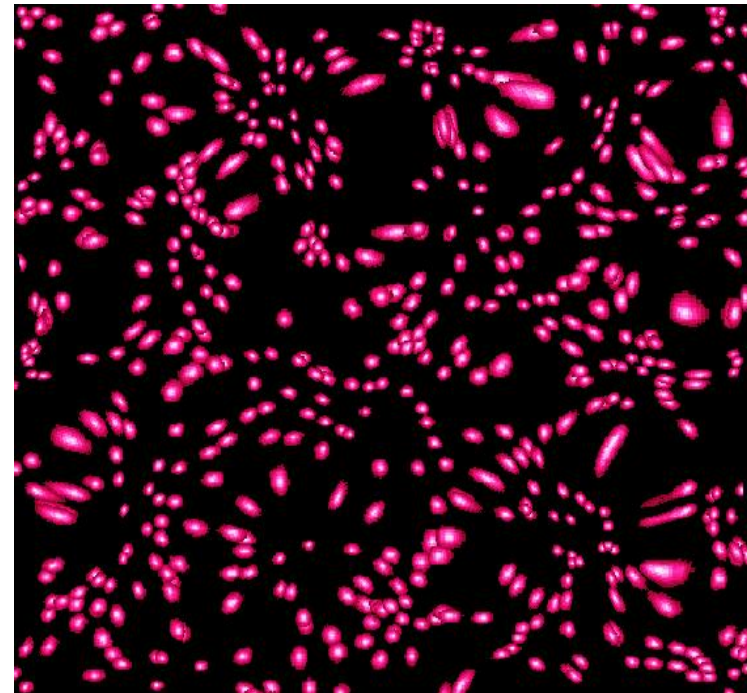
# Weak Lensing – A Powerful Probe for the Future



Distortion of background images by foreground matter



Unlensed



Lensed

---

FROM A WEAK LENSING SURVEY THE ANGULAR POWER SPECTRUM  
CAN BE CONSTRUCTED, JUST LIKE IN THE CASE OF CMB

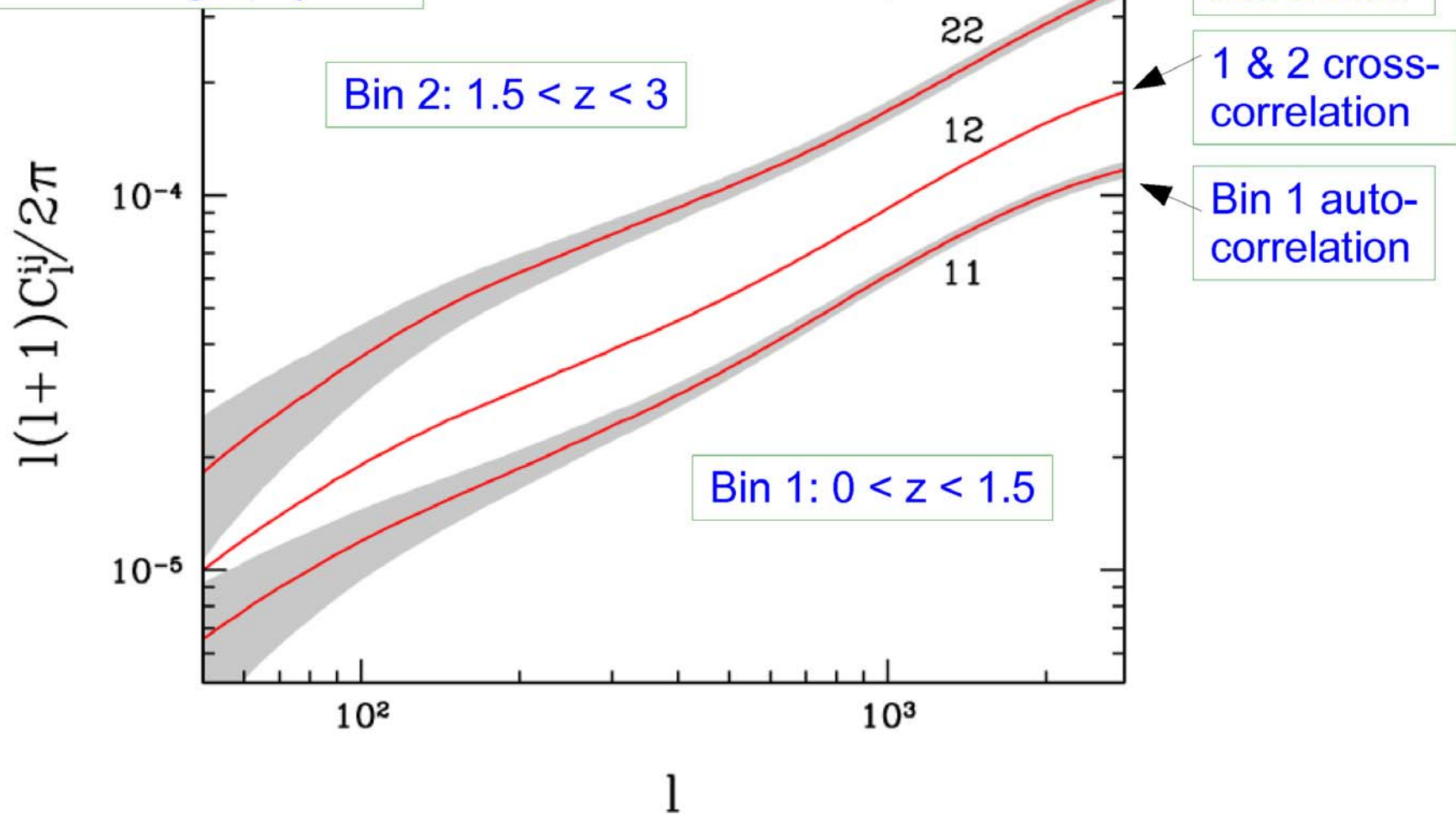
$$C_\ell = \frac{9}{16} H_0^4 \Omega_m^2 \int_0^{\chi_H} \left[ \frac{g(\chi)}{a\chi} \right]^2 P(\ell / r, \chi) d\chi$$

$P(\ell / r, \chi)$       MATTER POWER SPECTRUM (NON-LINEAR)

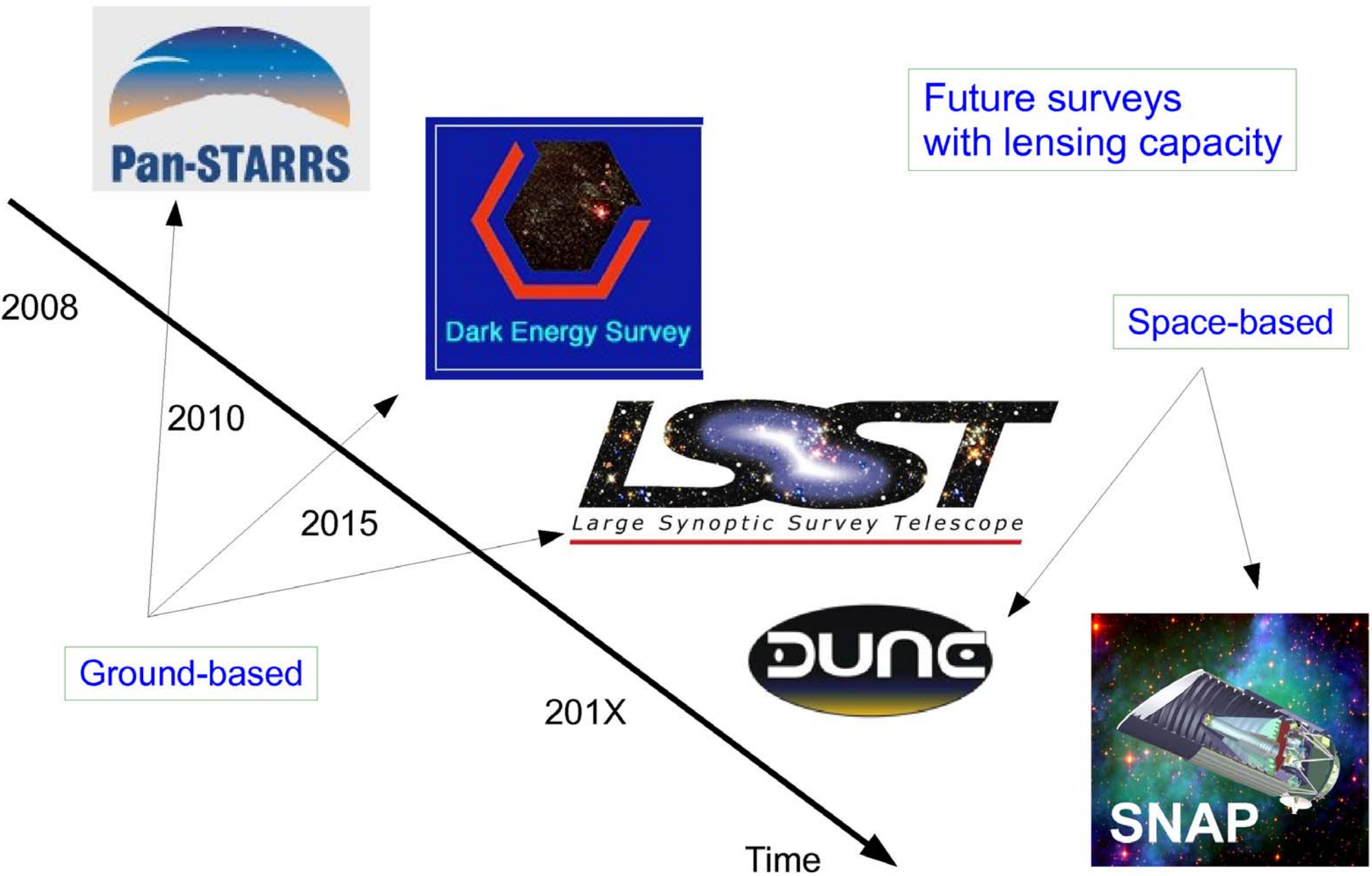
$$g(\chi) = 2 \int_0^{\chi_H} n(\chi') \frac{\chi(\chi' - \chi)}{\chi'} d\chi' \quad \text{WEIGHT FUNCTION  
DESCRIBING LENSING  
PROBABILITY}$$

(SEE FOR INSTANCE JAIN & SELJAK '96, ABAZAJIAN & DODELSON '03,  
SIMPSON & BRIDLE '04)

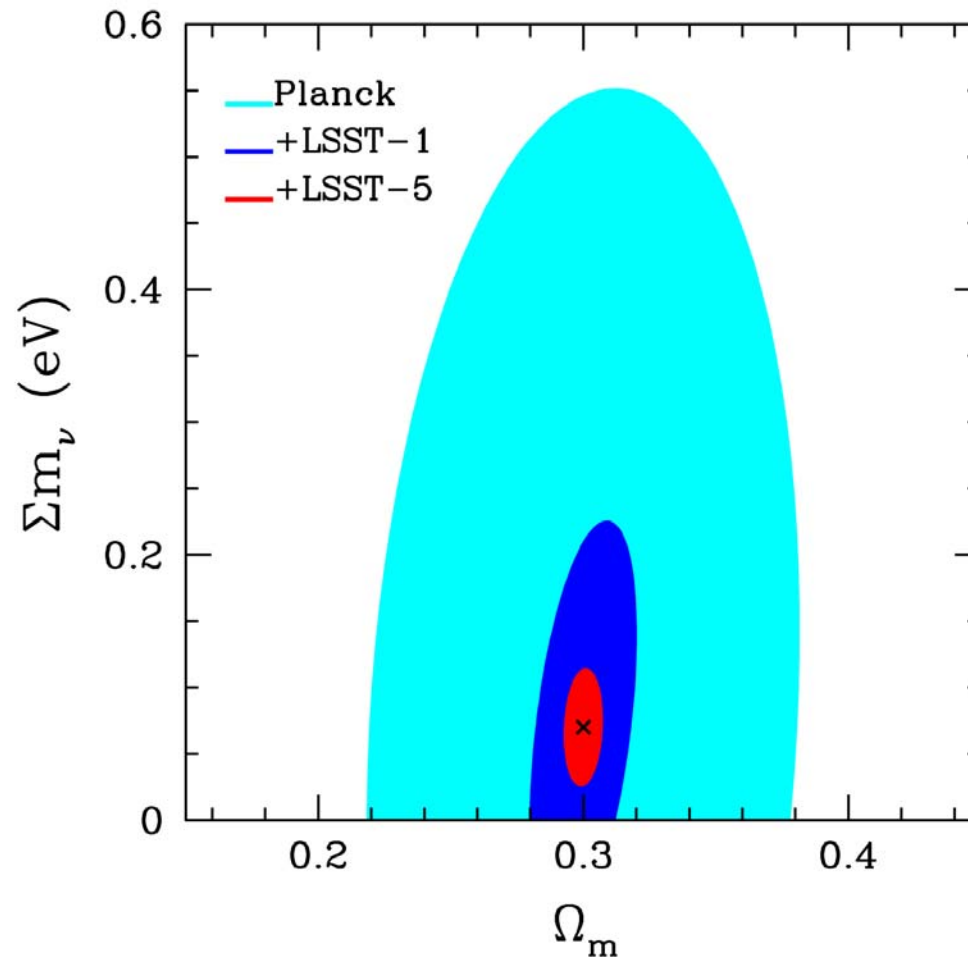
Shear power spectra  
for 2 tomography bins



STH, TU, WONG 2006



THE SENSITIVITY TO NEUTRINO MASS WILL IMPROVE TO  $< 0.1$  eV  
AT 95% C.L. USING WEAK LENSING  
COULD POSSIBLY BE IMPROVED EVEN FURTHER USING FUTURE  
LARGE SCALE STRUCTURE SURVEYS



STH, TU & WONG 2006 (ASTRO-PH/0603019, JCAP)

# Sensitivity Forecasts for Future LSS Observations

Lesgourgues, Pastor & Perotto, hep-ph/0403296	Planck & SDSS	$\Sigma m_\nu > 0.21$ eV detectable at $2\sigma$
	Ideal CMB & 40 x SDSS	$\Sigma m_\nu > 0.13$ eV detectable at $2\sigma$
Abazajian & Dodelson astro-ph/0212216	Future weak lensing survey 4000 deg <sup>2</sup>	$\sigma(m_\nu) \sim 0.1$ eV
Kaplinghat, Knox & Song, astro-ph/0303344	CMB lensing	$\sigma(m_\nu) \sim 0.15$ eV (Planck) $\sigma(m_\nu) \sim 0.044$ eV (CMBpol)
Wang, Haiman, Hu, Khoury & May, astro-ph/0505390	Weak-lensing selected sample of $> 10^5$ clusters	$\sigma(m_\nu) \sim 0.03$ eV
Hannestad, Tu & Wong astro-ph/0603019	Weak-lensing tomography (LSST plus Planck)	$\sigma(m_\nu) \sim 0.05$ eV

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# WHY IS WEAK LENSING TOMOGRAPHY SO GOOD?

IF MEASURED AT ONLY ONE REDSHIFT THE HOT DARK MATTER SIGNAL IS DEGENERATE WITH OTHER PARAMETERS

- CHANGING DARK ENERGY EQUATION OF STATE
- INITIAL CONDITIONS WITH BROKEN SCALE INVARIANCE

HOWEVER, BY MEASURING AT DIFFERENT REDSHIFTS THIS DEGENERACY CAN BE BROKEN

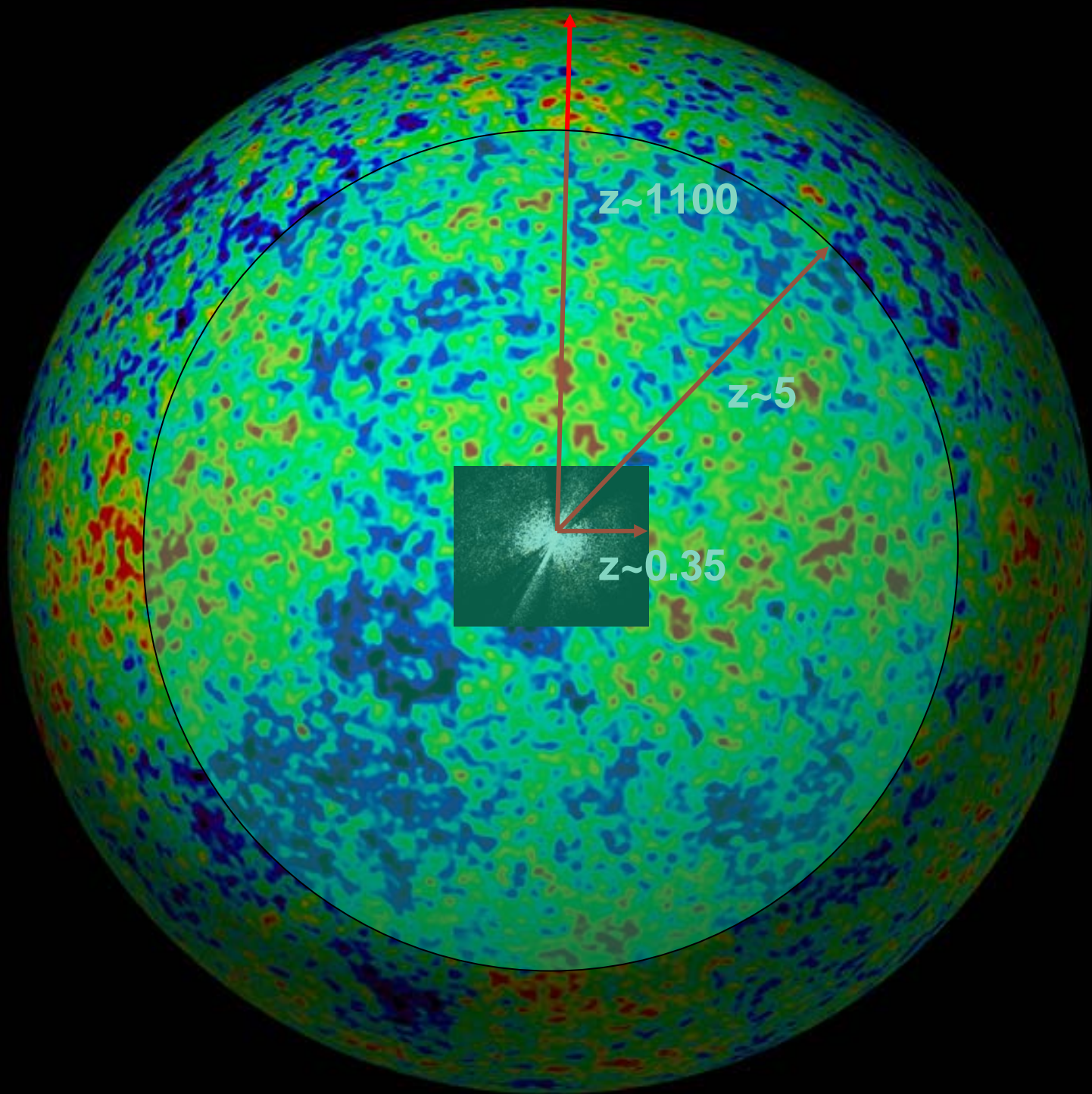
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## ADVANTAGES OF PROBING STRUCTURE AT HIGH REDSHIFT

- STRUCTURES ARE MORE LINEAR
- VOLUME IS LARGER (THE HORIZON VOLUME IS  $\sim 1000$  TIMES LARGER THAN THE SDSS-LRG VOLUME)

### HOW TO DO IT?

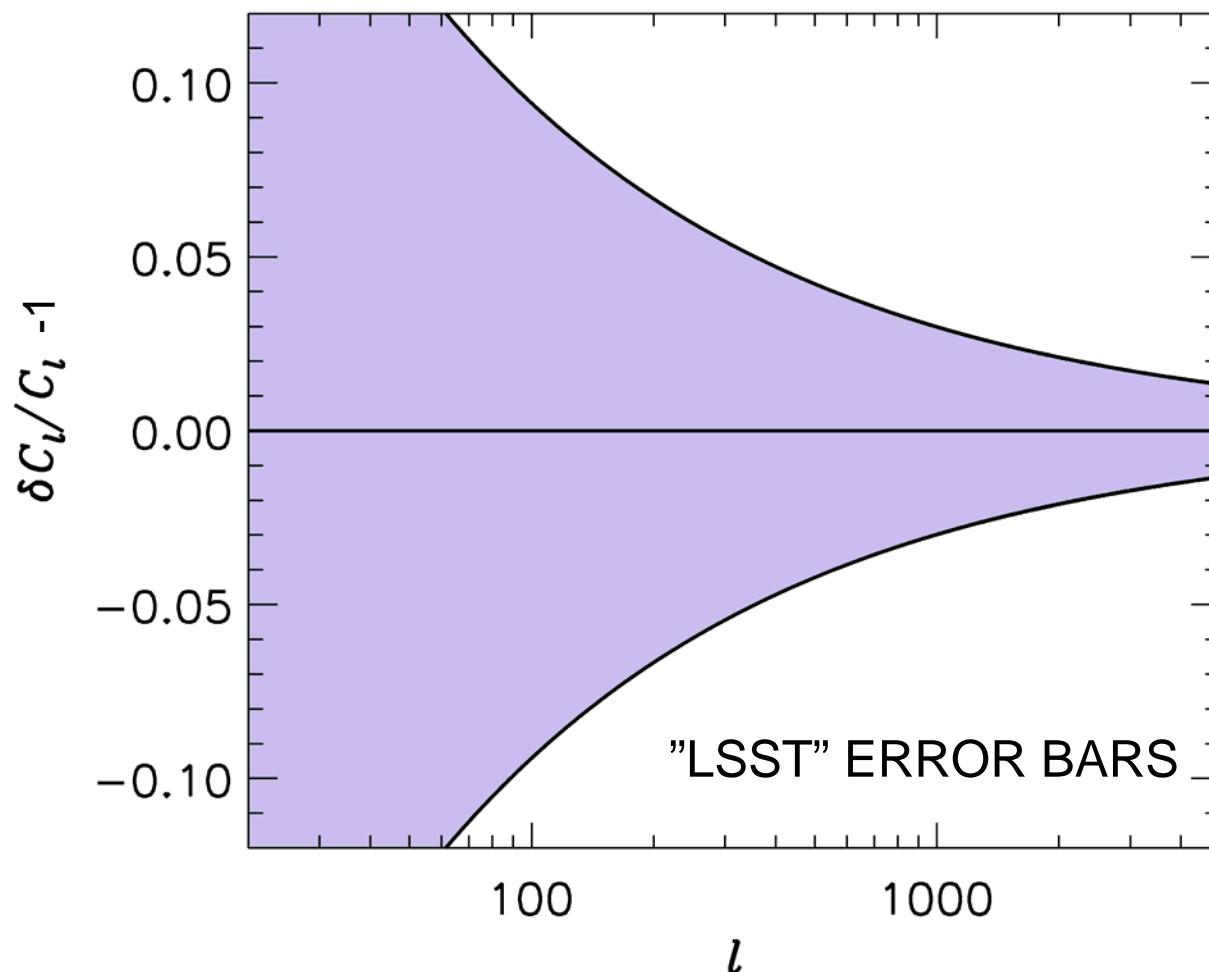
- LYMAN-ALPHA?
- BAO? (WF MOS)
- GALAXIES? (LSST)
- 21-CM? (SKA)



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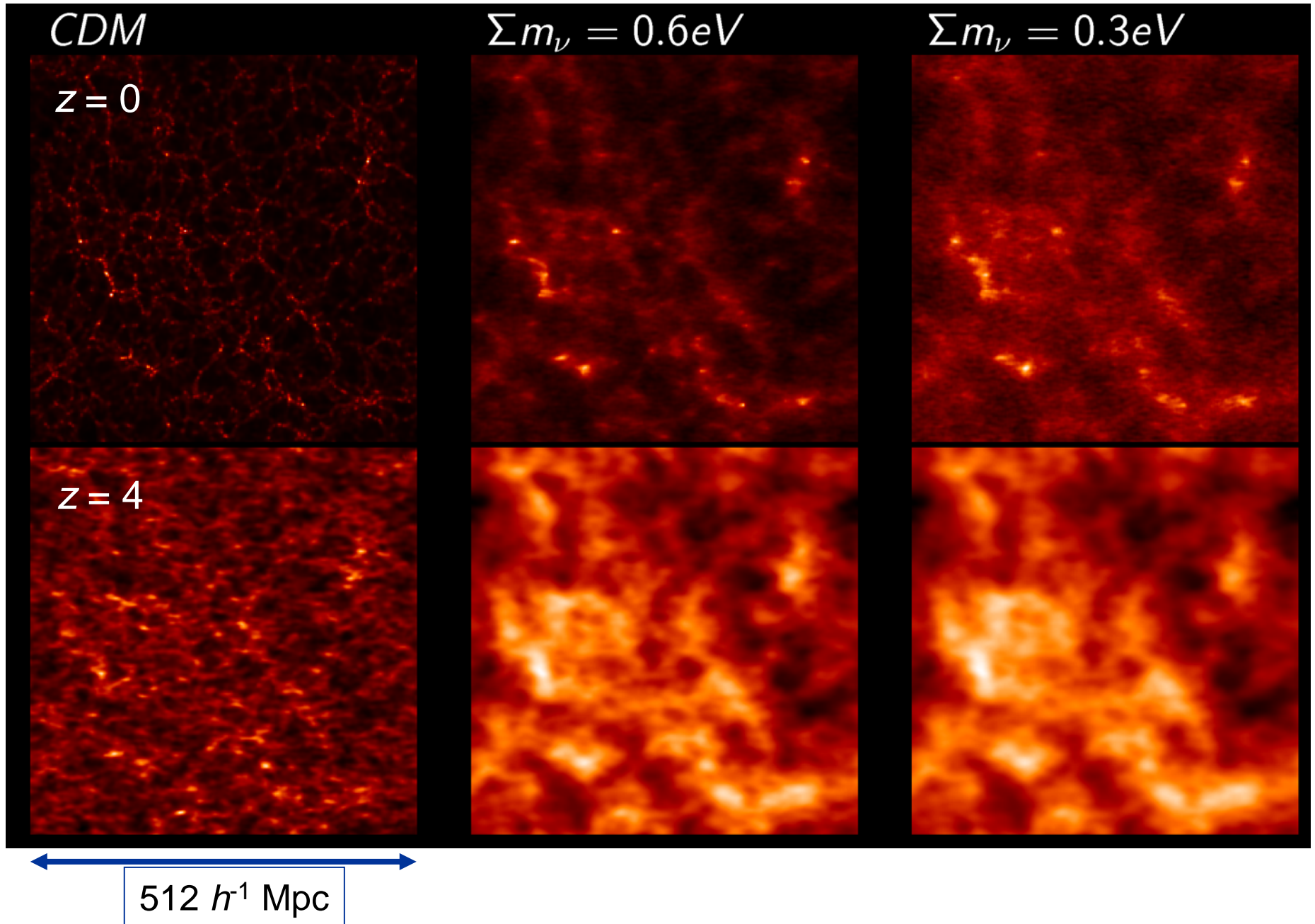
THIS SOUNDS GREAT, BUT UNFORTUNATELY THEORY MUST BE AT  
THE SAME LEVEL OF PRECISION BY THE TIME DATA-TAKING STARTS

FUTURE SURVEYS LIKE LSST WILL PROBE THE POWER SPECTRUM  
TO  $\sim 1$ -2 PERCENT PRECISION

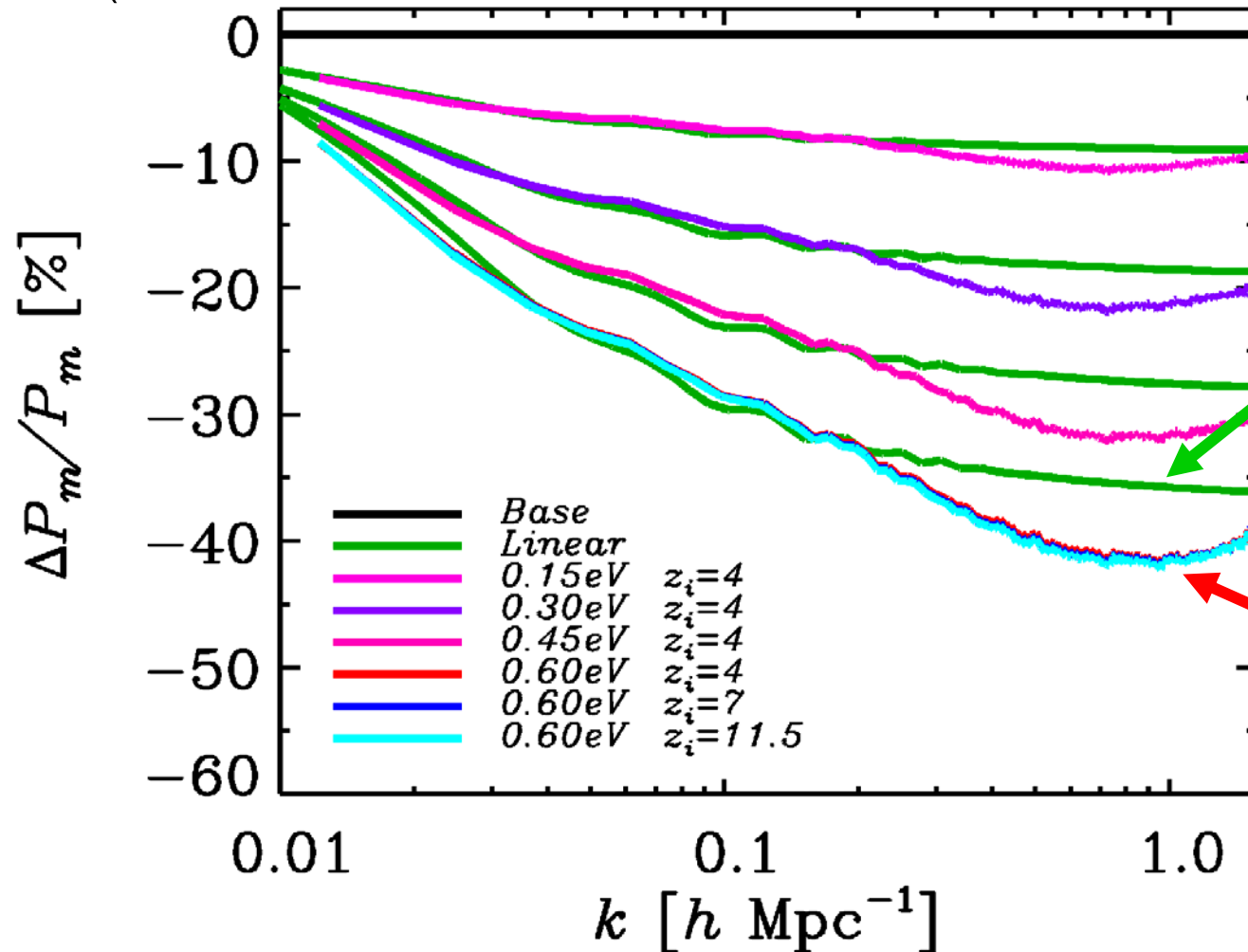


WE SHOULD BE ABLE TO CALCULATE THE POWER SPECTRUM  
TO AT LEAST THE SAME PRECISION!

# Example: Evolution of neutrino density field



NON-LINEAR EVOLUTION PROVIDES AN ADDITIONAL AND VERY CHARACTERISTIC SUPPRESSION OF FLUCTUATION POWER DUE TO HDM (COULD BE USED AS A SMOKING GUN SIGNATURE)



LINEAR THEORY

$$\frac{\Delta P}{P} \sim -8 \frac{\Omega_v}{\Omega_m}$$

FULL NON-LINEAR

$$\frac{\Delta P}{P} \sim -9.6 \frac{\Omega_v}{\Omega_m}$$

Brandbyge, STH, Haugbølle, Thomsen, arXiv:0802.3700 (ApJ)

# Summary

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Cosmology provide very competitive laboratory for testing axion physics

Measurements of the large scale structure of the universe can potentially probe both very low and very high mass axions

In the coming years there will be a large amount of new and very precise cosmological data which will push the axion bounds