Photon Axion Conversions in transversely inhomogeneous Magnetic Fields:

A primer

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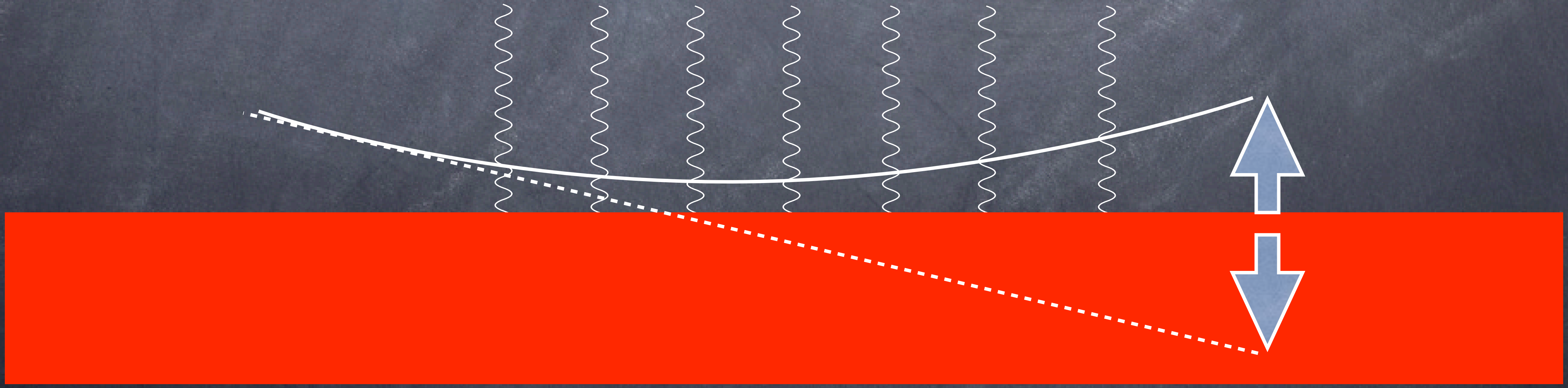
5th Axion-WIMP-WISP workshop, DURHAM

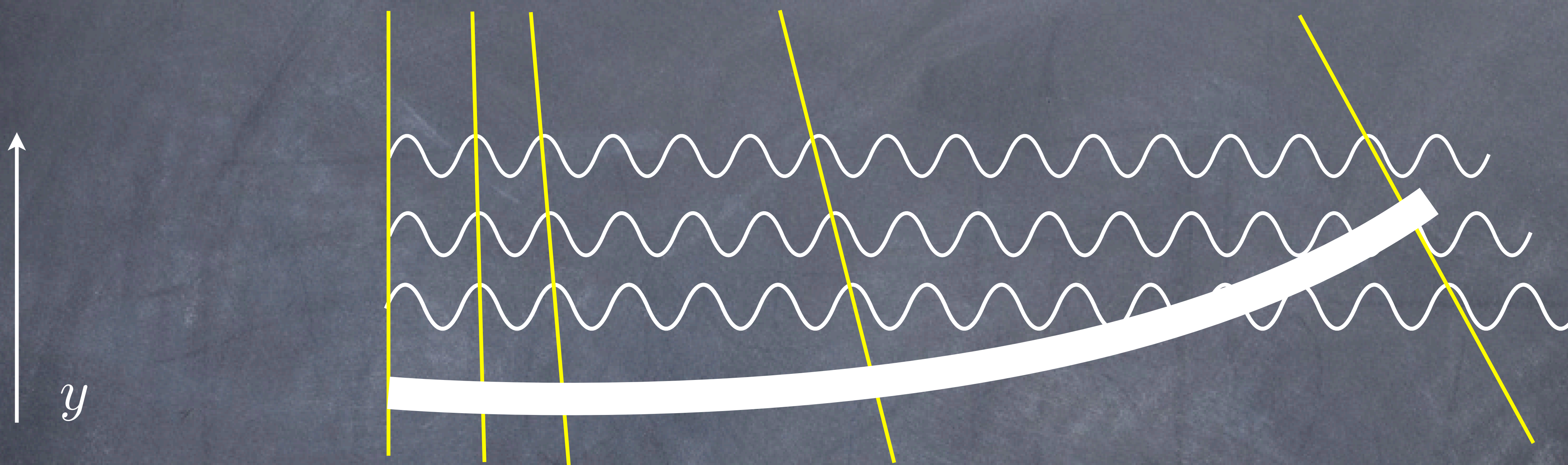
(in collaboration with Joerg Jaeckel)

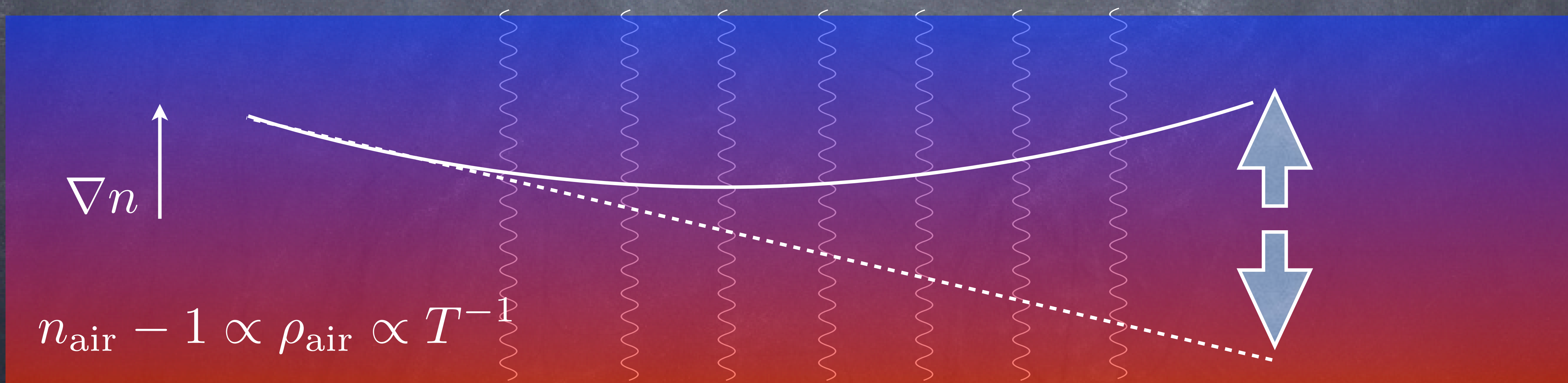
Invitation: Mirages



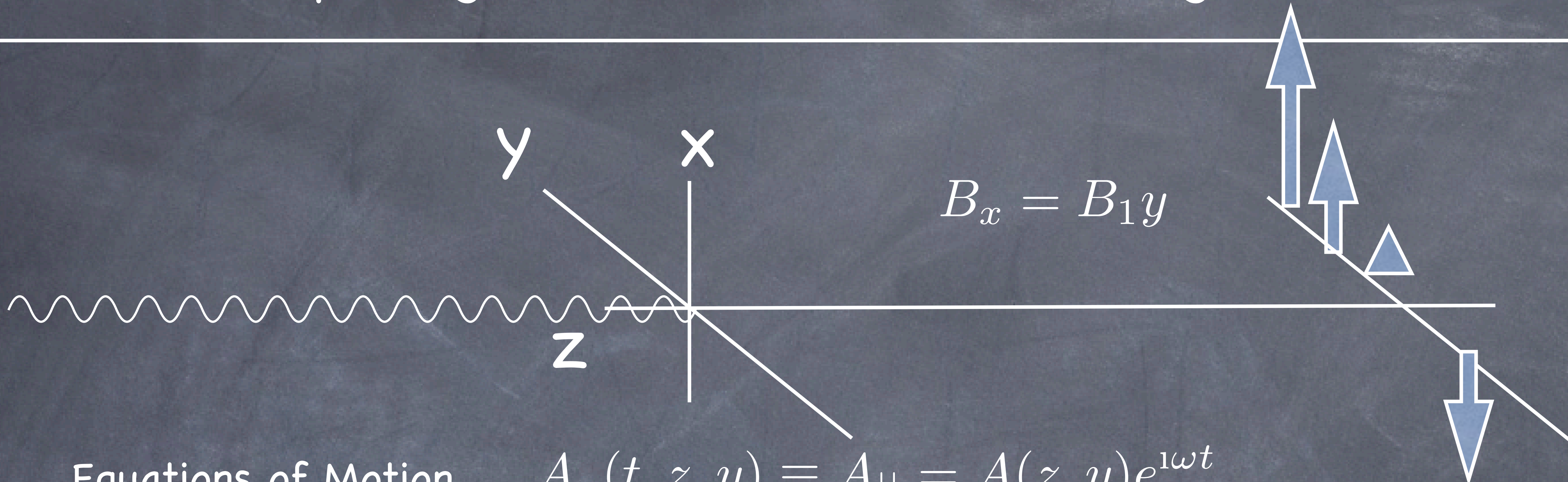
Invitation

$n = n(y)$ $\qquad$ $k = k(y) = \omega\, n(y)$





Photon Axion Splitting in a transverse Gradient Magnetic Field



Equations of Motion $\quad A_x(t, z, y) \equiv A_{||} = A(z, y)e^{\imath\omega t}$

$$(\omega^2 + \nabla^2)\begin{pmatrix} A \\ a \end{pmatrix} - \begin{pmatrix} 0 & gB\omega \\ gB\omega & \cancel{m_a^2} \end{pmatrix}\begin{pmatrix} A \\ a \end{pmatrix} = 0$$

Eigenstates of propagation $\quad A_{\pm} = \dfrac{A \pm a}{\sqrt{2}}$

$(\omega^2 + \nabla^2)A_{\pm} = \pm g B_x \omega \longrightarrow$ Index of refraction $\quad n_{\pm}^2 = 1 \mp \dfrac{g B_x(y)}{\omega}$

$$\nabla n_{\pm} \simeq \mp \frac{g}{2\omega} \nabla B_x = \mp \frac{g B_1}{2\omega}$$

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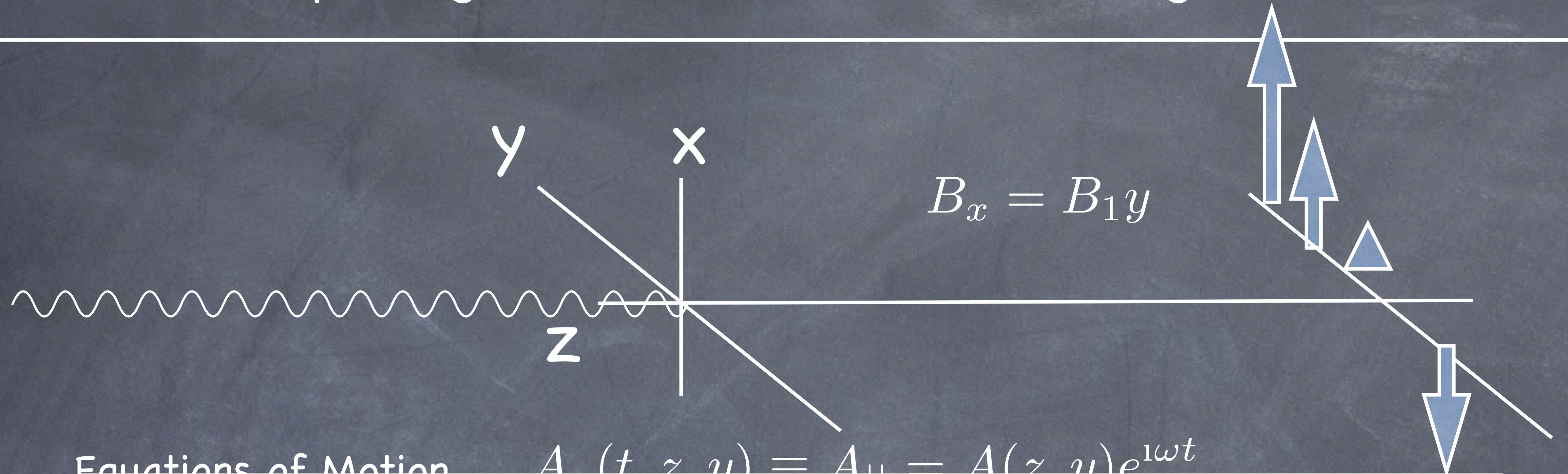

Equations of Motion $A_{\pm}(t, z, y) = A_{\pm} = A(z, y)e^{i\omega t}$

The gradients of the refraction index of the + and - waves have opposite sign, they curve in different directions

Eigenstates of propagation

$$A_{\pm} = \frac{A \pm a}{\sqrt{2}}$$

$$(\omega^2 + \nabla^2)A_{\pm} = \pm g B_x \omega \longrightarrow$$

Index of refraction

$$n_{\pm}^2 = 1 \mp \frac{g B_x(y)}{\omega}$$

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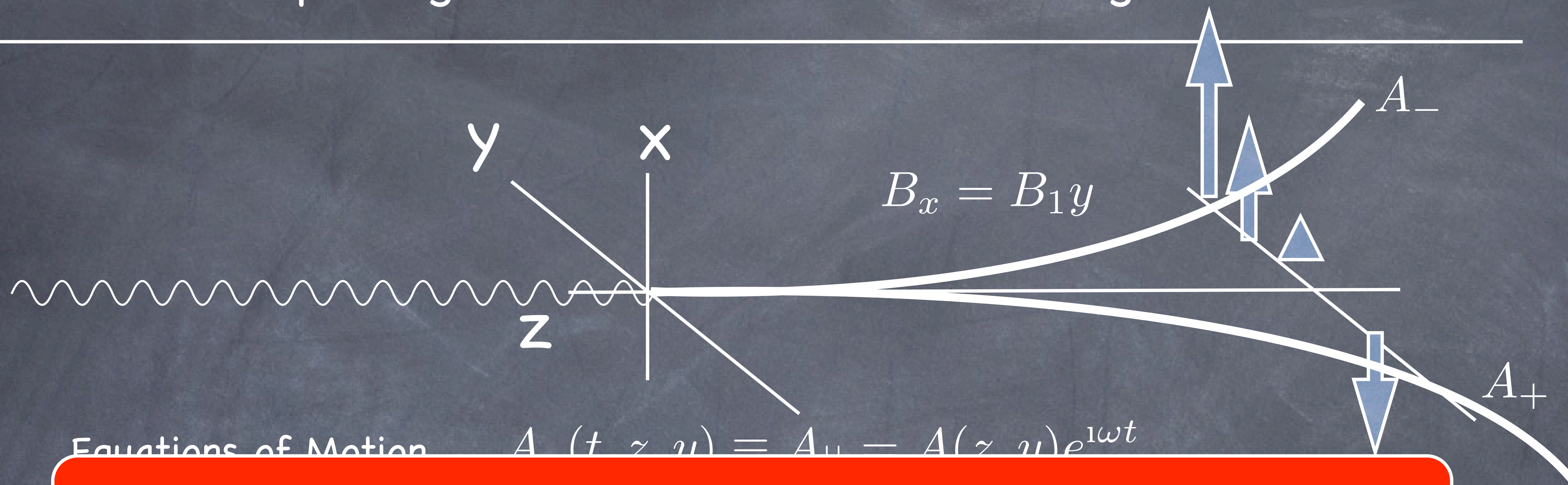

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Phase Fronts

Constant Magnetic Field $B_x = B_0 \neq f(y)$

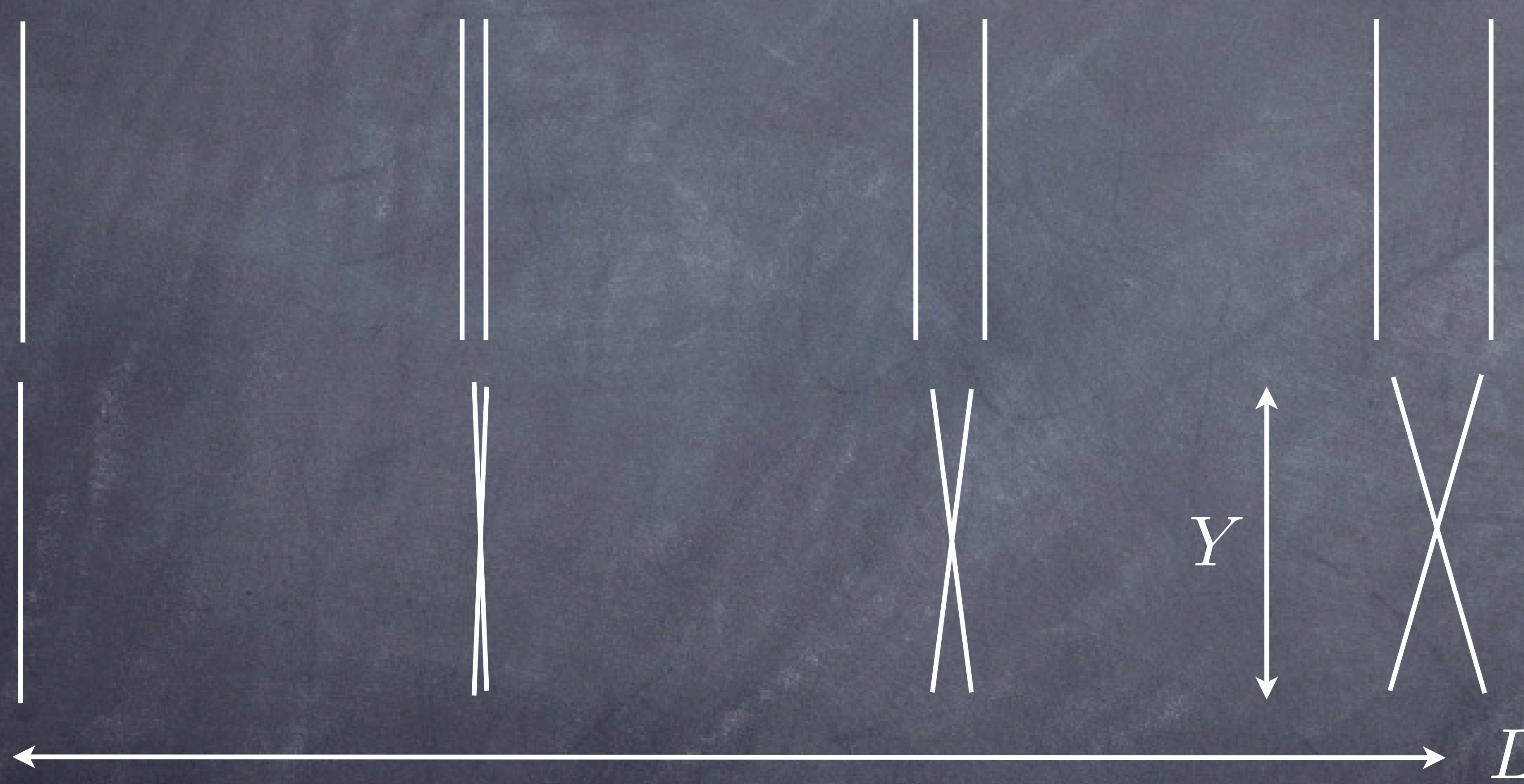
$$(\omega^2 + \nabla^2)A_\pm = \pm g B_x \omega \qquad k_\pm \simeq \omega \mp \frac{gB_x}{2}$$

Gradient Magnetic Field (naively) $B_x = B_1 y$

Probability for an extended wave front

Very naively ...

$$P(A \to a) = \ldots \left| e^{\imath\phi_+} - e^{\imath\phi_-} \right|^2 = \sin^2 \frac{\phi_+ - \phi_-}{2}$$



$$\phi_+ - \phi_- = \frac{L}{\lambda} \frac{g B_0}{\omega}$$

$$\langle \phi_+ - \phi_- \rangle = \frac{L}{\lambda} \frac{1}{2} \frac{g B_1 Y}{2\omega}$$

For an experimental setup design:
If you are limited by the maximum magnetic field

$$B_0 \sim B_1 Y$$

(Loose a factor of 4 in the conversion probability...)

Calculation in the eikonal approximation

$$(\omega^2 + \nabla^2)A_\pm = \pm g B_x \omega \qquad (\omega^2 + \nabla^2)A = g B_x \omega$$

$$A_x = e^{\imath\omega t} A(y,z) = \mathcal{A}(y,z) e^{\imath\omega t} e^{-\imath\omega S(y,z)}$$

slowly varying

EIKONAL

fast varying (phase fronts)

$$(\nabla S)^2 - \imath\frac{1}{\omega}\frac{\nabla(\mathcal{A}^2\nabla S)}{\mathcal{A}^2} - \frac{1}{\omega^2}\frac{\nabla^2\mathcal{A}}{\mathcal{A}^2} = 1 - \frac{gB_x\omega}{\omega^2} \equiv n^2(y) \equiv 1 - \mathcal{B}y$$

$$(\nabla S)^2 = n^2(y)$$

Hamilton equations (method of characteristics) $\vec{p}(s) = \nabla S(\vec{r}(s))$

$$\frac{d\vec{r}}{ds} = \vec{p} \quad ; \quad \frac{d\vec{p}}{ds} = \vec{\nabla} n^2/2 \ .$$

Calculation in the eikonal approximation

$$(\nabla S)^2 = n^2(y)$$

Hamilton equations (method of characteristics) $\vec{p}(s) = \nabla S(\vec{r}(s))$

Ray Solutions $\quad \dfrac{d\vec{r}}{ds} = \vec{p} \quad ; \quad \dfrac{d\vec{p}}{ds} = \vec{\nabla} n^2/2 \; .$

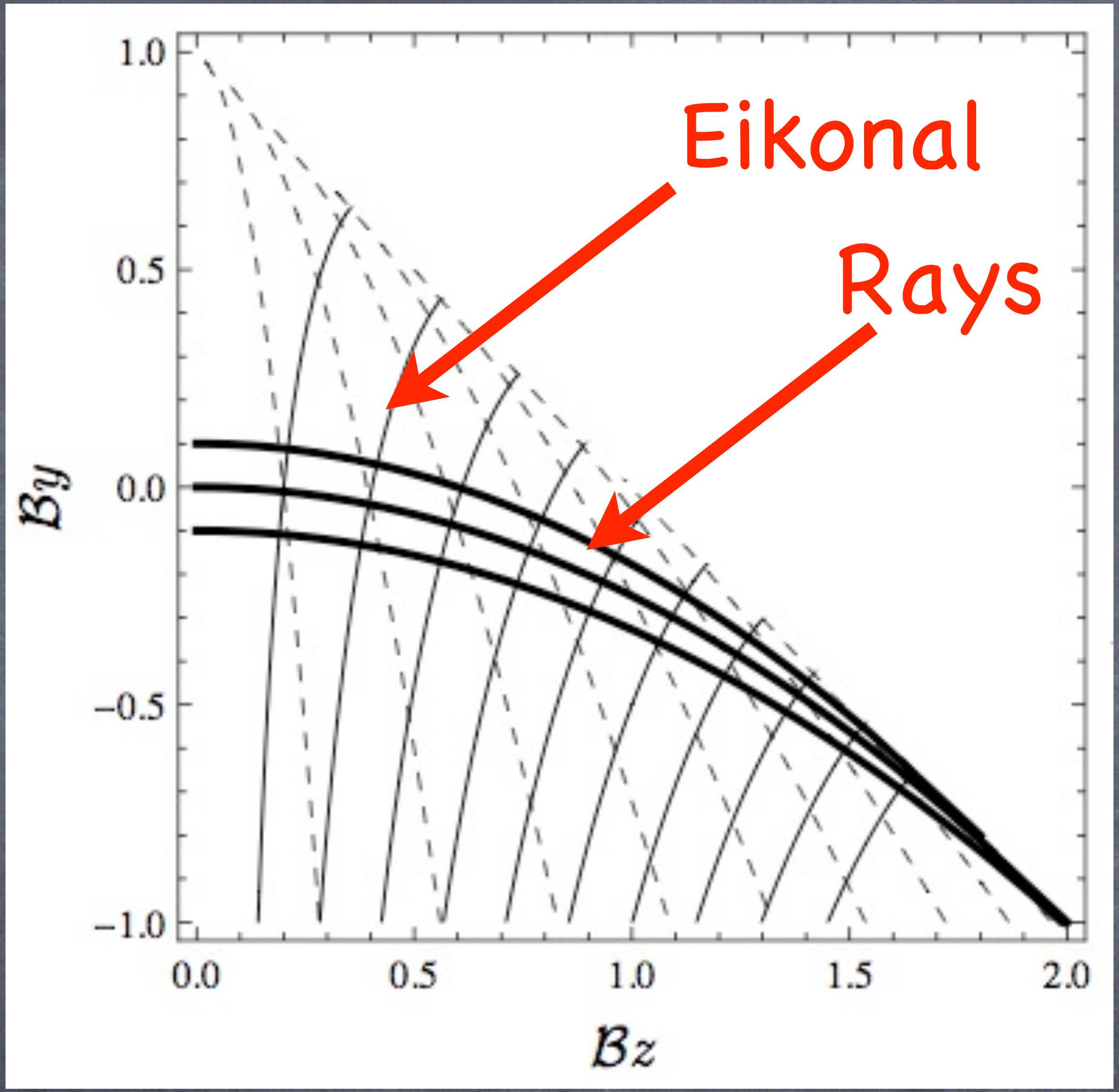
$$\vec{r}(s, y_0) = \begin{pmatrix} y(s) \\ z(s) \end{pmatrix} = \begin{pmatrix} y_0 - \mathcal{B}s^2/4 \\ sn(y_0) \end{pmatrix} \quad ; \quad \vec{p}(s, y_0) = \begin{pmatrix} -\mathcal{B}s/2 \\ n(y_0) \end{pmatrix},$$

parabolae

$$S(s, y_0) = \int_s^s \vec{\nabla} S \cdot d\vec{r}(s) = \int_0^s \vec{p}(s', y_0) \cdot \frac{d\vec{x}(s', y_0)}{ds} ds' = sn^2(y_0) + \frac{\mathcal{B}^2 s^3}{12}$$

$$S(y, z) = zn(y) \left(\frac{2}{1 + \sqrt{1 - \xi^2}} \right)^{1/2} \left(\frac{2 + \sqrt{1 - \xi^2}}{3} \right), \quad \xi = \frac{\mathcal{B} z}{n^2(y)}$$

Calculation in the eikonal approximation



Relevant length scale

$$z_{\mathcal{B}} = \mathcal{B}^{-1} = \frac{\omega}{gB_1} \simeq 5 \times 10^{16}\mathrm{m}\left(\frac{\omega}{\mathrm{eV}}\right)\left(\frac{g}{10^{-10}\mathrm{GeV}^{-1}}\right)^{-1}\left(\frac{B_1}{\mathrm{T/m}}\right)^{-1}$$

Probability in the EIKONAL approximation

$$\frac{d}{dy}P(\gamma_{||}\to a) = \frac{1}{4}|A_+ - A_-|^2 = \left|e^{-\imath\omega S_+} - e^{-\imath\omega S_-}\right|^2 = \sin^2\frac{\omega\Delta S}{2}$$

$$\Delta S = S(\mathcal{B}) - S(-\mathcal{B}) = S(y) - S(-y)$$

Expand around $\mathcal{B}y = 0$

$$\Delta S(y,z) \simeq z\mathcal{B}y\left(\frac{2}{1+\sqrt{1-(\mathcal{B}z)^2}}\right)^{1/2} \equiv z\mathcal{B}y \times f(\mathcal{B}z)$$

$$f \in (1, \sqrt{2})$$

Compare with the constant case $B_x = B_0 \equiv B_1 Y$

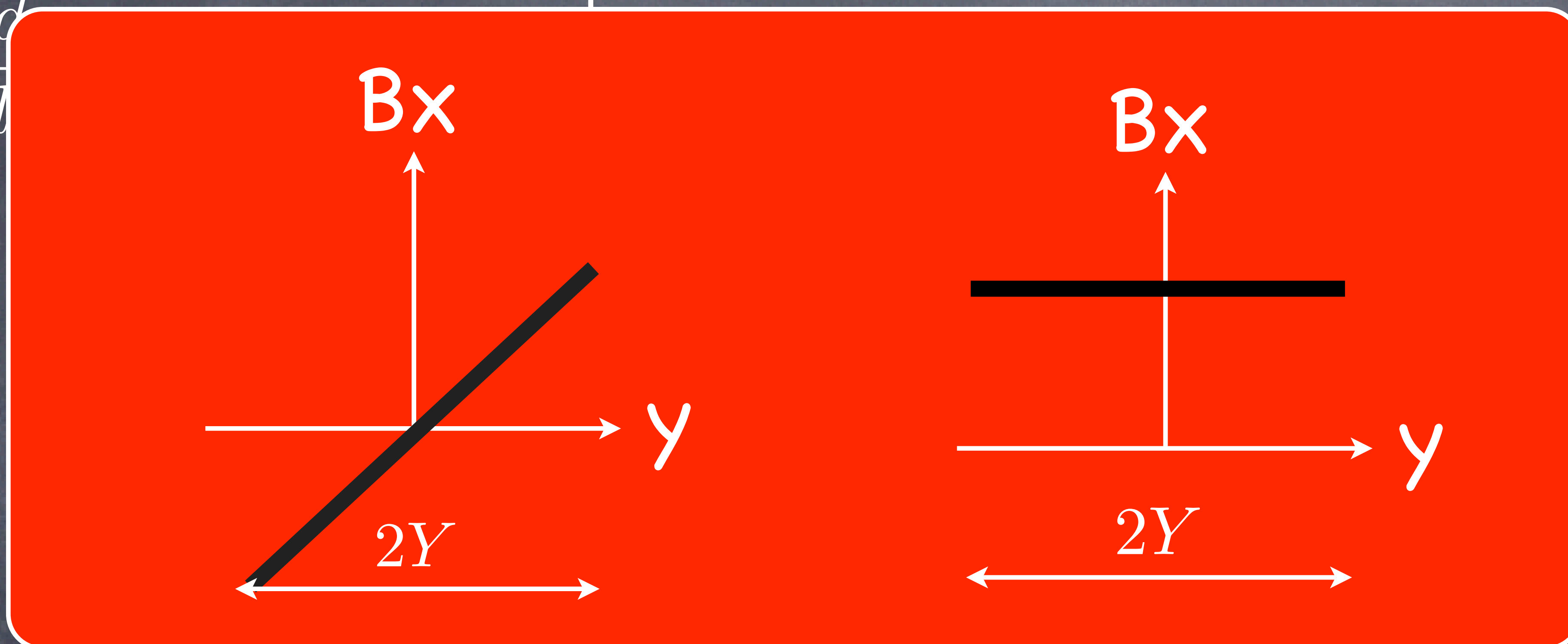
$$\frac{1}{2Y}\int_{-Y}^{Y} P_{\gamma\to a}(B = B_1 y)dy = \frac{1}{2}\left(1 - \mathrm{sinc}\left(\omega z\mathcal{B}Y f\right)\right) \simeq (\omega z\mathcal{B}Y)^2\frac{f^2}{12}$$

$$\frac{1}{2Y}\int_{-Y}^{Y} P^{\mathrm{std}}_{\gamma\to a}(B = B_1 Y)dy = \sin^2\left(\frac{\omega z\mathcal{B}Y}{2}\right) \simeq \frac{1}{4}\left(\omega z\mathcal{B}Y\right)^2$$

$$\omega z\mathcal{B}Y = g(B_1 Y)z$$

Probability in the EIKONAL approximation

$$\frac{d}{d} \ldots {}^{2} = \sin^2 \frac{\omega \Delta S}{2}$$



$$f(\mathcal{B}z)$$

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$$\omega z \mathcal{B} Y = g(B_1 Y) z$$

Conclusions

- Photon-Axion spliting is there (A+, A- splitting indeed)

- Relevant length scale for ray curvature is huge

$$z_{\mathcal{B}} = \mathcal{B}^{-1} = \frac{\omega}{gB_1} \simeq 5 \times 10^{16}\mathrm{m} \left(\frac{\omega}{\mathrm{eV}}\right) \left(\frac{g}{10^{-10}\mathrm{GeV}^{-1}}\right)^{-1} \left(\frac{B_1}{\mathrm{T/m}}\right)^{-1}$$

- Expressions for the conversion probability (for lab sizes) are similar to those in a constant transverse field (B0) with

$$B_0 \sim B_1 Y$$

Gradient x transverse size

- Exact calculation in the Eikonal approximation for a plane wave

- Also performed calculations for Gaussian beams (relevant for laser experiments)

THANK YOU !!